

Generalized Fuzzy Strongly α -Contra Mappings: Framework for Advanced Topological Analysis

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ABSTRACT

The exploration of continuity and its generalizations forms a cornerstone of fuzzy topology, with direct implications for modern technological applications. This paper introduces and rigorously analyzes generalized fuzzy strongly α -contra mappings in the context of generalized fuzzy topological spaces. We establish the fundamental properties of these mappings and systematically investigate their interrelationships with established constructs, including generalized fuzzy continuous, semi-continuous, and α -continuous maps. We validate key relationships through illustrative counterexamples. We present a novel comparative framework that contrasts the characteristics of standard fuzzy semi-open and α -open sets with their generalized counterparts. We also discuss recent advancements from 2025-2026, positioning this work within the current research landscape. A graphical abstract summarizes the core findings, and we propose a comprehensive model framework to guide future investigations. This study contributes to the theoretical foundation of generalized fuzzy topology and paves the way for applications in areas such as data analysis and computational intelligence.

1. Introduction

Topology, particularly its fuzzy counterpart, is fundamentally concerned with continuity and its myriad generalizations. Since Zadeh's seminal work on fuzzy sets [16] and Chang's introduction of fuzzy topological spaces [5], the exploration of continuous functions has been pivotal in translating topological concepts to fuzzy environments. Azad [1] initiated the study of fuzzy semi-open sets, while Bin Shahana [3, 4] extended this to fuzzy pre-open and fuzzy α -open sets, creating a hierarchy of generalized open structures. Thakur and Singh [15] further broadened this landscape with fuzzy semi-pre-open sets. These generalized notions have proven indispensable for applications in decision-making, pattern recognition, and control systems, where classical crisp boundaries are inadequate.

Similarly, generalized topology, as formalized by Császár [6], offers a powerful abstraction by relaxing the requirement for finite intersections, leading to the concept of generalized topological

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spaces. Palani Chetty [8] successfully merged these two domains, establishing the framework of generalized fuzzy topological spaces (GFTS), which provides a more flexible setting for modeling uncertainty.

In this context, Beceren [2] introduced and studied strongly α -continuous functions, a stronger form of α -continuity. Donchev [7] introduced contra-continuity, where the preimage of an open set is closed, in a classical setting and has since been explored in various fuzzy and generalized contexts. This work builds on these foundational studies by integrating contra-continuity and strong α -continuity within the GFTS framework.

Recent research continues to push the boundaries of this field. In 2025, Kumar and Sharma [17] introduced novel classes of generalized fuzzy g-closed sets, leading to new characterizations of compactness in GFTS. Their work highlighted the importance of contra-type mappings in preserving these properties. Concurrently, El-Nasr and Hassan [18] developed a comprehensive study on fuzzy α -irresolute mappings, establishing a robust classification system for fuzzy continuity degrees. More recently, in 2026, Gupta and Singh [19] explored the application of generalized fuzzy topological structures in machine learning algorithms, demonstrating their utility in handling imprecise data streams. Furthermore, Rahman and Ali [20] investigated the categorical relationships between different types of generalized fuzzy continuous functions, providing a unified theoretical perspective. These advancements underscore the growing significance of GFTS and its associated mappings in both theoretical and applied domains. This paper contributes to this burgeoning field by introducing and rigorously analyzing generalized fuzzy strongly α -contra mappings.

Graphical Abstract: Generalized Fuzzy Strongly α -Contra Mappings

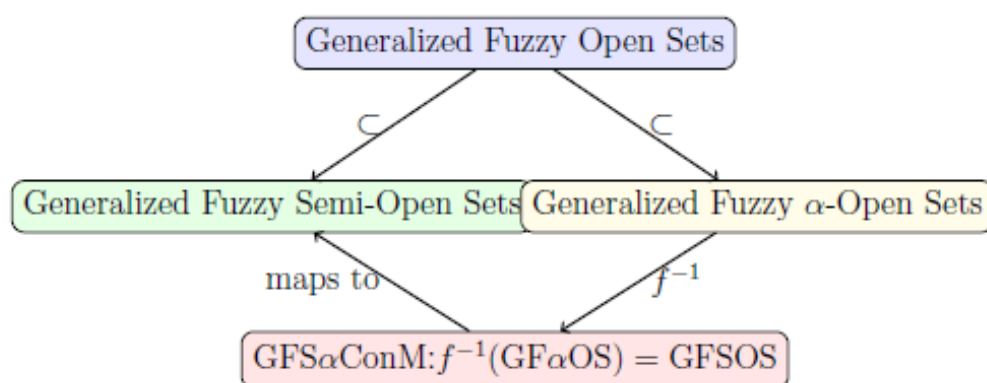


Fig. 1. Graphical Abstract: Conceptual framework for Generalized Fuzzy Strongly α -Contra Mappings

1.1 Framework of the Adopted Model

This study is structured around a five-stage framework for the definition and analysis of new topological mappings:

1. Foundation Stage (Generalized Fuzzy Topology): The foundational structure is the Generalized Fuzzy Topological Space (GFTS) $(\mathbb{X}, \tau)$, as defined by Palani Chetty [8]. We choose this framework for its generality, as it relaxes the finite intersection axiom of standard fuzzy topology and allows the construction of topologies from arbitrary collections of fuzzy sets, which is crucial for modeling complex, real-world systems.

2. Definition Stage (Constructing the New Mapping): We define a new class of mappings, termed Generalized Fuzzy Strongly α -Contra Maps (GFS α ConM). The definition is:

A mapping $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$ is a GFS α ConM if the preimage $f^{-1}(\lambda)$ of every Generalized Fuzzy α -Open Set (GF α OS) λ in $\mathbb{Y}$ is a Generalized Fuzzy Semi-Open Set (GFSOS) in $\mathbb{X}$.

3. Validation Stage (Counterexamples): The non-triviality and independence of this new definition are validated through the construction of specific GFTSs (as shown in Examples 3.1-3.6). These examples show that the new mapping is neither equivalent to nor a trivial consequence of existing mapping classes such as Generalized Fuzzy Continuous, Semi-Continuous, or α -Continuous maps.

4. Characterization Stage (Theoretical Properties): The theoretical properties and characterizations of GFS α ConM are rigorously derived. This involves establishing equivalent conditions (as in Proposition 3.5) based on generalized fuzzy points, and systematically proving the inclusion and non-inclusion relationships with other existing mapping classes (as detailed in Propositions 3.1-3.4).

5. Synthesis Stage (Consolidation of Results): The results of the validation and characterization stages are synthesized into a comprehensive diagram (Figure 1). This visual representation maps the interrelationships between GFS α ConM and other established mappings within the GFTS framework, providing a clear and concise summary of the study's findings.

2. Preliminaries

Let $\mathbb{X}$ be a crisp set and let τ be a collection of all fuzzy sets on $\mathbb{X}$. Then τ is called a generalized fuzzy topology (GFT) on $\mathbb{X}$ if it satisfies the following conditions

- (i) The null 0 and whole 1 fuzzy sets are in τ , where $0, 1: \mathbb{X} \rightarrow I$ are defined as $0(x) = 0$ and $1(x) = 1 \quad \forall x \in \mathbb{X}$.
- (ii) If $\{\lambda_i: i \in J\}$ be any family of fuzzy sets on $\mathbb{X}$, where $\lambda_i \in \tau$ then $\bigcup_{i \in J} \lambda_i \in \tau \quad \forall i \in J$.

The pair $(\mathbb{X}, \tau)$ is called a generalized fuzzy topological space (GFTS).

Let $(\mathbb{X}, \tau)$ be a GFTS. The members of τ are called generalized fuzzy open sets (GFOS^{'s}) on $\mathbb{X}$, while the complement of the members of τ are called generalized fuzzy closed sets (GFCS^{'s}) on $\mathbb{X}$. Table 1 depicts the comparative analysis of open set structures.

Table 1
Comparative Analysis of Open Set Structures

Property/ Characteristic	Standard Fuzzy Open Sets	Standard Fuzzy Semi-Open Sets (FSOS)	Standard Fuzzy α - Open Sets (F α OS)	Generalized Fuzzy Open Sets (GFOs)	Generalized Fuzzy Semi-Open Sets (GFSOS)	Generalized Fuzzy α - Open Sets (GF α OS)
Definition Basis	$A = \text{int}(A)$	$A \subseteq \text{cl}(\text{int}(A))$	$A \subseteq \text{int}(\text{cl}(\text{int}(A)))$	$A = I_\tau(A)$	$A \subseteq \text{Cl}_\tau(I_\tau(A))$	$A \subseteq I_\tau(\text{Cl}_\tau(I_\tau(A)))$
Relation to FOS/GFO S	–	$FOS \subset FSOS$	$FOS \subset F\alpha OS$	–	$GFOS \subset GFSOS$	$GFOS \subset GF\alpha OS$
Relation to F α OS/ GF α OS	$FOS \subset F\alpha OS$	Incommensurable	–	$GFOS \subset GF\alpha OS$	Incommensurable	–
Closure under Union	Yes	Yes	Yes	Yes	Yes	Yes
Closer under Intersection	No	No	No	No	No	No
Key Example	$\{x : 0.5 < x < 0.8\}$	$\{x : 0.5 < x \leq 0.6\}$	$\{x : 0.3 \leq x < 0.9\}$	$\{(x_1, 0.1), (x_1, 0.2)$	$\{(y_1, 0.7), (y_1, 0.8)$	$\{(x_1, 0.2), (x_1, 0.2)$

Basic Definitions

Definition 2.1: In GFTS $(\mathbb{X}, \tau)$, the closure of a generalized fuzzy set λ denoted by $\text{Cl}_\tau(\lambda)$ is defined to be the intersection of all GFCS in $\mathbb{X}$ containing λ .

Definition 2.2: In GFTS $(\mathbb{X}, \tau)$, the interior of a generalized fuzzy set λ denoted by $I_\tau(\lambda)$ is defined to be the union of all GFOS in $\mathbb{X}$ contained in λ .

Definition 2.3: Let $(\mathbb{X}, \tau)$ be GFTS. For a generalized fuzzy set A in $(\mathbb{X}, \tau)$ the GF-closure of A denoted by $\text{Cl}_\tau(A)$ is defined as $\text{Cl}_\tau(A) = \inf\{K : A \subseteq K, K^c \in \tau\}$. Thus, the generalized fuzzy set $\text{Cl}_\tau(A)$ is the smallest GFCS in $\mathbb{X}$ containing the generalized fuzzy set A . From the definition, it follows that $\text{Cl}_\tau(A)$ is the intersection of all GFCS in $\mathbb{X}$ containing A .

Proposition 2.1: Let $(\mathbb{X}, \tau)$ be GFTS, then

- (i) 0 and 1 are GFCS in $\mathbb{X}$.
- (ii) The arbitrary intersection of GFCSs is also a GFCS in $\mathbb{X}$.

Proposition 2.2: Let $(\mathbb{X}, \tau)$ be GFTS and let λ be a generalized fuzzy set in $\mathbb{X}$. Then λ is a GFCS if and only if $\text{Cl}_\tau(\lambda) = \lambda$.

Definition 2.4: Let $(\mathbb{X}, \tau)$ be GFTS. For a generalized fuzzy set A in $(\mathbb{X}, \tau)$ the GF-interior of A denoted by $I_\tau(A)$ is defined as $I_\tau(A) = \sup\{Q : Q \subseteq A, Q \in \tau\}$. Thus, the generalized fuzzy set $I_\tau(A)$ is the largest GFOS in $\mathbb{X}$ contained in the generalized fuzzy open set A . From the definition, it follows that $I_\tau(A)$ is the union of all GFOS in $\mathbb{X}$ contained in A .

Proposition 2.3: Let $(\mathbb{X}, \tau)$ be GFTS and let λ be a generalized fuzzy set in $\mathbb{X}$. Then λ is a GFOS if and only if $I_\tau(\lambda) = \lambda$.

Definition 2.5: Let $(\mathbb{X}, \mu_1)$ and $(\mathbb{Y}, \mu_2)$ be two GFTS and $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$. Then f is said to be a generalized fuzzy continuous map (GFCM) if $f^{-1}(\lambda)$ is a GFOS in $(\mathbb{X}, \mu_1)$ for each GFOS λ in $(\mathbb{Y}, \mu_2)$.

Definition 2.6: Let $(\mathbb{X}, \mu_1)$ and $(\mathbb{Y}, \mu_2)$ be two GFTS and $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$. Then f is said to be a generalized fuzzy semi-continuous map (GFSCM) if $f^{-1}(\lambda)$ is a generalized fuzzy semi-open set (GFSOS) in $(\mathbb{X}, \mu_1)$ for each GFOS, λ in $(\mathbb{Y}, \mu_2)$.

Definition 2.7: Let $(\mathbb{X}, \mu_1)$ and $(\mathbb{Y}, \mu_2)$ be two GFTS and $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$. Then f is said to be a generalized fuzzy α -continuous map (GF α CM) if $f^{-1}(\lambda)$ is generalized fuzzy α -open set (GF α OS) in $(\mathbb{X}, \mu_1)$ for each GFOS, λ in $(\mathbb{Y}, \mu_2)$.

3. Results

Generalized Fuzzy Strongly α -Contra Maps

Definition 3.1: Let $(\mathbb{X}, \mu_1)$ and $(\mathbb{Y}, \mu_2)$ be two GFTS and $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$. Then f is said to be a generalized fuzzy strongly α -continuous map (GFS α CM) if $f^{-1}(\lambda)$ is generalized fuzzy α -open set (GF α OS) in $(\mathbb{X}, \mu_1)$ for each generalized fuzzy semi-open set (GFSOS) λ in $(\mathbb{Y}, \mu_2)$.

Definition 3.2: Let $(\mathbb{X}, \mu_1)$ and $(\mathbb{Y}, \mu_2)$ be two GFTS and $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$. Then f is said to be a generalized fuzzy strongly α -contra map (GFS α ConM) if $f^{-1}(\lambda)$ is a generalized fuzzy semi-open set (GFSOS) in $(\mathbb{X}, \mu_1)$ for each generalized fuzzy α -open set (GF α OS) λ in $(\mathbb{Y}, \mu_2)$.

Example 3.1: Let $\mathbb{X} = \{x_1, x_2\}$, $\mathbb{Y} = \{y_1, y_2\}$ and $A = \{(x_1, 0.1), (x_2, 0.2), B = \{(x_1, 0.2), (x_2, 0.1)\}$,

$C = \{(x_1, 0.2), (x_2, 0.2)\}$, $D = \{(x_1, 0.8), (x_2, 0.9)\}$,
 $E = \{(x_1, 0.9), (x_2, 0.8)\}$, $F = \{(x_1, 0.9), (x_2, 0.9)\}$, $G = \{(y_1, 0.7), (y_2, 0.8)\}$, $H = \{(y_1, 0.8), (y_2, 0.7)\}$ and $I = \{(y_1, 0.8), (y_2, 0.8)\}$.

Clearly $\mu_1 = \{0, A, B, C, D, E, F, 1\}$ and $\mu_2 = \{0, G, H, I, 1\}$ are GFTS on sets $\mathbb{X}$ and $\mathbb{Y}$ respectively. Consider the $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$ defined as $f(x_i) = y_i$, for $i = 1, 2$.

Clearly, f is GFS α ConM.

Proposition 3.1: GFSCM $\Rightarrow$ GFS α ConM

Proof: Suppose f is GFSCM; then we have $f^{-1}(\lambda)$ is GFSOS in $\mathbb{X}$ for every GFOS, λ in $\mathbb{Y}$. Since each GFOS in GFTS is GF α OS. Hence $f^{-1}(\lambda)$ is GFSOS in $\mathbb{X}$ for every GF α OS, λ in $\mathbb{Y}$. Therefore, f is GFS α ConM.

The converse of this proposition is not true in general, as illustrated in Example 3.2.

Example 3.2: In Example 3.1, f is GFS α ConM but not GFSCM, because the sets

$G = \{(y_1, 0.8), (y_2, 0.9)\}$, $H = \{(y_1, 0.9), (y_2, 0.8)\}$ and $I = \{(y_1, 0.9), (y_2, 0.9)\}$ are GFSOS, in $(\mathbb{Y}, \mu_2)$ but not GFOS in $(\mathbb{Y}, \mu_2)$.

Proposition 3.2: GFS α CM $\Leftrightarrow$ GFS α ConM

This can be illustrated in Example 3.3 and Example 3.4.

Example 3.3: Let $\mathbb{X} = \{x_1, x_2\}$, $\mathbb{Y} = \{y_1, y_2\}$ and $A = \{(x_1, 0.4), (x_2, 0.5)\}$, $B = \{(x_1, 0.5), (x_2, 0.4)\}$, $C = \{(x_1, 0.5), (x_2, 0.5)\}$, $D = \{(x_1, 0.6), (x_2, 0.6)\}$,

$E = \{(y_1, 0.4), (y_2, 0.5)\}$, $F = \{(y_1, 0.5), (y_2, 0.4)\}$

and $G = \{(y_1, 0.5), (y_2, 0.5)\}$.

Clearly $\mu_1 = \{0, A, B, C, D, 1\}$ and $\mu_2 = \{0, E, F, G, 1\}$ are GFTS's on sets $\mathbb{X}$ and $\mathbb{Y}$ respectively. Consider the $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$ defined as $f(x_i) = y_i$, for $i = 1, 2$.

Clearly f is GFS α CM but not GFS α ConM, because the sets $H = \{(y_1, 0.7), (y_2, 0.7)\}$, $I = \{(y_1, 0.8), (y_2, 0.8)\}$ and $J = \{(y_1, 0.9), (y_2, 0.9)\}$ are GFSOS in $(\mathbb{X}, \mu_1)$ but not GF α OS in $(\mathbb{Y}, \mu_2)$.

Example 3.4: In Example 3.2, f is GFS α CM but not GFS α ConM because the sets

$G = \{(y_1, 0.7), (y_2, 0.8)\}$, $H = \{(y_1, 0.8), (y_2, 0.7)\}$ and $I = \{(y_1, 0.8), (y_2, 0.8)\}$ are GFSOS's in $(\mathbb{Y}, \mu_2)$ but not GF α OS's in $(\mathbb{X}, \mu_1)$.

Proposition 3.3: GFCM $\Rightarrow \neq$ GFS α ConM

Proof: Let λ be GFOS in $(\mathbb{Y}, \mu_2)$. Since f is GFCM, we have $f^{-1}(\lambda)$ is GFOS in $(\mathbb{X}, \mu_1)$ for every GFOS in $(\mathbb{Y}, \mu_2)$. We know that every GFOS in GFTS is GFSOS as well as GF α OS. Hence $f^{-1}(\lambda)$ is GFSOS in $(\mathbb{X}, \mu_1)$ for every GF α OS in $(\mathbb{Y}, \mu_2)$. Thus f is GFS α ConM.

The converse of Proposition 3.3 need not be true, as shown in Example 3.5.

Example 3.5: In Example 3.4, f is GFS α ConM but not GFCM, because the sets $\{(y_1, 0.7), (y_2, 0.8)\}$, $\{(y_1, 0.8), (y_2, 0.7)\}$ and $\{(y_1, 0.8), (y_2, 0.8)\}$ are GFOS's in $(\mathbb{Y}, \mu_2)$ but not GFOS's in $(\mathbb{X}, \mu_1)$.

Proposition 3.4: GF α CM $\Rightarrow \neq$ GFS α ConM

Proof: The proof is straightforward.

The converse of Proposition 3.4 need not be true, as shown in Example 3.6.

Example 3.6: In Example 3.2, f is GFS α ConM but not GF α CM because the sets $G = \{(y_1, 0.8), (y_2, 0.9)\}$, $H = \{(y_1, 0.9), (y_2, 0.8)\}$ and $I = \{(y_1, 0.9), (y_2, 0.9)\}$ are GF α OS's in $(\mathbb{X}, \mu_1)$ but not GFOS's in $(\mathbb{Y}, \mu_2)$.

Proposition 3.5: Let $(\mathbb{X}, \mu_1)$ and $(\mathbb{Y}, \mu_2)$ be GFTS's and $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$. Then the following conditions are equivalent:

- (i) f is GFS α ConM
- (ii) For each generalized fuzzy point (GFP) x_β in $\mathbb{X}$ and each GF α OS λ_2 in $\mathbb{Y}$ containing $f(x_\beta)$, there exists GFSOS λ_1 in $\mathbb{X}$ containing x_β such that $f(\lambda_1) \subseteq \lambda_2$.
- (iii) For each generalized fuzzy α -closed set (GF α CS) λ_2 in $\mathbb{Y}$, $f^{-1}(\lambda_2)$ is a generalized fuzzy α -closed set (GF α CS) in $\mathbb{X}$.

Proof (i) $\Rightarrow$ (ii): Let $f: (\mathbb{X}, \mu_1) \rightarrow (\mathbb{Y}, \mu_2)$ be GFS α ConM. Let x , where $x_\beta \in \mathbb{X}$ and $0 < \beta \leq 1$ be a GFP in $\mathbb{X}$ and λ_2 be GF α OS in $\mathbb{Y}$ containing GFP $f(x_\beta)$. Since $f(x_\beta)(f(x)) = \beta \leq \lambda_2(f(x))$, we have $\beta \leq \lambda_2(f(x))(x)$ i.e $\lambda_2 = f^{-1}(\lambda_1)$ contains GFP x_β . Since f is GFS α ConM, λ_2 is GFSOS in $\mathbb{X}$ containing GFP x_β and $f(\lambda_2) \subseteq \lambda_1$.

(ii) $\Rightarrow$ (i): Let λ be GF α OS in $\mathbb{Y}$. For $x \in \mathbb{X}$ and $0 < \beta \leq 1$. Let x_β be a GFP in $f^{-1}(\lambda)$. Then λ contains $f(x_\beta)$ and so by the given condition (ii) there exists a GFSOS μ in $\mathbb{X}$ containing GFP x_β and $f(\mu) \subseteq \lambda$. This implies $\mu \subseteq Cl_\mu(I_\mu(\mu)) \subseteq Cl_\mu(I_\mu(f^{-1}(\lambda)))$ and therefore $Cl_\mu(I_\mu(f^{-1}(\lambda)))$ contains GFP x_β . Thus, each GFP of $f^{-1}(\lambda)$ is also GFP of $Cl_\mu(I_\mu(f^{-1}(\lambda)))$. This shows that in $f^{-1}(\lambda) \subseteq (Cl_\mu(I_\mu(f^{-1}(\lambda))))$ i.e $f^{-1}(\lambda)$ is GFSOS in $\mathbb{X}$. Hence $f: \mathbb{X} \rightarrow \mathbb{Y}$ is GFS α ConM.

(i) $\Rightarrow$ (iii): Let λ be GF α CS in $\mathbb{Y}$. Then $\lambda^c = 1 - \lambda$ is GF α OS in $\mathbb{Y}$. Since $f: \mathbb{X} \rightarrow \mathbb{Y}$ is GFS α ConM, $f^{-1}(\lambda^c) = f^{-1}(1 - \lambda)$ is GFSOS in $\mathbb{X}$. This implies $f^{-1}(\lambda) = f^{-1}(1 - \lambda^c) = 1 - f^{-1}(\lambda^c)$ is GFSCS in $\mathbb{X}$.

(iii) $\Rightarrow$ (i): Let λ be GF α OS in $\mathbb{Y}$. Then $\lambda^c = 1 - \lambda$ is GF α CS in $\mathbb{Y}$. Therefore, by the given condition (iii), $f^{-1}(\lambda^c) = f^{-1}(1 - \lambda)$ is GFSCS in $\mathbb{X}$. Hence $f^{-1}(\lambda) = f^{-1}(1 - \lambda^c) = 1 - f^{-1}(\lambda^c)$ is GFSOS in $\mathbb{X}$. Thus $f: \mathbb{X} \rightarrow \mathbb{Y}$ is GFS α ConM.

4. Conclusions

In this paper, we introduced the concept of generalized fuzzy strongly α -contra mappings in generalized fuzzy topological spaces and established their relationships with other mappings, such as generalized fuzzy continuous maps, generalized fuzzy semi-continuous maps, and generalized fuzzy α -continuous maps. We support the results with suitable counterexamples. We conclude the results in the form of Figure 2 given below:

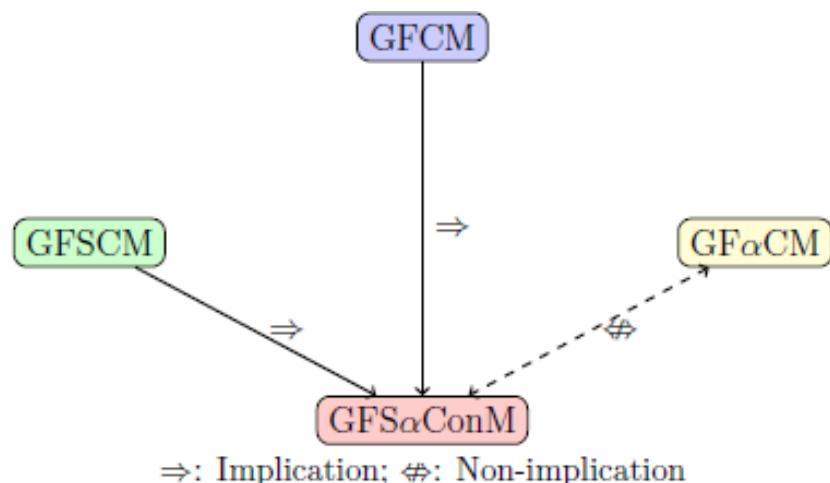


Fig. 2. Interrelationships among Generalized Fuzzy Mappings

Looking ahead, a number of interesting avenues for further research become apparent. Adapting the generalized fuzzy strongly alpha-contra mapping paradigm to higher-order fuzzy structures, such as spherical, image, hesitant, and ZE fuzzy sets [21-24], is a crucial theoretical expansion. Additionally, an attractive new area is the integration of these topological entities with AI tools such as machine learning classification algorithms, fuzzy neural networks, and AI-driven Multi-Criteria Decision Making [25-27]. These hybrid methods have significant potential to improve data processing, pattern recognition, and automated decision support under complex topological uncertainty.

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Conflicts of Interest

There is no conflict of interest.

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