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An Interval-Valued Multi-Criteria Decision-Making Framework for Flexible Manufacturing System Selection under Uncertainty

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ABSTRACT

Choosing a suitable flexible manufacturing system (FMS) is an important multi-criteria decision-making (MCDM) problem, which involves uncertain conditions and conflicting criteria. In this paper, we have suggested an interval valued MCDM method to solve FMS selection problem in uncertain environment. Interval valued numbers have been used to capture uncertainty in criteria values and eliminate the limitations associated with precise values. Our suggested method comprises center-, lower-, upper- and radius-based representation of intervals along with interval extensions of some distance-based MCDM methods such as Combinative Distance-Based Assessment (CODAS), Evaluation Based on Distance from Average Solution (EDAS) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). An illustrative FMS selection problem is considered to demonstrate the usefulness of our proposed approach. Results reveal that all the interval valued MCDM methods give similar ranking of alternatives for various representations of intervals, which demonstrates the robustness and applicability of the method. In addition, performance analysis of average ranks and rank stability shows the consistency of the selected best alternative.

1. Introduction

The Multi-Criteria Decision-Making (MCDM) approach is increasingly gaining popularity as a widely used approach to solving complicated decision problems, which usually have more than one and sometimes even contradictory criteria. This approach allows for an integrated assessment of different alternatives in the light of qualitative and quantitative information available through the use of some decision-making procedure. Due to its great flexibility and efficiency, the MCDM approach has been successfully applied in various fields such as production systems, logistics, environmental planning, engineering design, healthcare, transportation, and business management [1,2]. The general problem of MCDM involves selecting the set of possible alternatives, determining the criteria for evaluation, allocating proper weights for these criteria, and combining the performance of alternatives through proper decision models like the Analytic Hierarchy Process (AHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), VIKOR, Evaluation Based on Distance from Average Solution (EDAS), and Combinative Distance-Based Assessment (CODAS). The process of ranking alternatives in terms of their performance through these models is

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essential for decision-making in engineering and management fields [10,14,15,38,39]. In the wide range of industrial MCDM applications, the problem of selecting the right Flexible Manufacturing System (FMS) is regarded as one of the most difficult. The contemporary manufacturing industries exist in highly competitive and dynamic environments where productivity, flexibility, quality, effectiveness, and efficiency play a crucial role in staying competitive. The FMS allows manufacturing companies to be ready to adjust to changes in demand and shorten the time of manufacturing products. However, the process of FMS selection requires taking into consideration a large number of conflicting and uncertain criteria at the same time [10,14,20]. When it comes to real-world FMS selection problems, the decision-making criteria usually consist of not only the beneficial criteria, where higher values are favored (such as quality, adaptability, flexibility, productivity, and expandability), but also non-beneficial criteria, where lower values are preferable (for example, maintenance cost, depreciation, energy cost, and production cost). The concurrent analysis of such diverse decision criteria considerably adds to the difficulty of making decisions. Therefore, the proper aggregation techniques and weight setting approaches are needed for combining the decision criteria [2,7]. In general, conventional MCDM approaches require that the criterion values and the weights assigned to these be expressed in terms of numbers. In practice, however, decision makers often face uncertainties caused by factors such as partial data, imprecise judgment, measurement errors, expert opinion, and varying working conditions. Expressing uncertain data by means of exact numbers can sometimes cause misleading or incorrect decision-making results. The use of intervals is an efficient way of incorporating uncertain data into decision-making by assigning an interval value to each criterion value instead of using a single number [6,9,11,23]. Interval mathematics has thus attracted much interest in the field of decision making due to its capability to accommodate uncertainties without any need for making probabilistic assumptions. Various scholars have modified traditional MCDM approaches with the use of interval information in order to make decisions more reliable in uncertain conditions. Interval valued TOPSIS is one such method for ranking alternatives by utilizing interval positive and negative ideal solutions [11,16], whereas interval EDAS and CODAS have proved their merit in applications in the field of engineering and industry [38,39,42]. Similarly, interval valued MOORA, MULTIMOORA, VIKOR, and several other aggregation methods have been effectively used in the evaluation of manufacturing systems and similar optimization problems [27,43,44]. Recent research has resulted in intelligent decision-making frameworks that utilize computational methods to enhance the accuracy, adaptability, and robustness of the process of multi-criteria decision making in a complex environment [52]. In spite of the considerable progress made, there are still some major deficiencies that need to be addressed. The overwhelming majority of the available researches considers the application of only one interval-valued MCDM method or compares no more than a few approaches. Moreover, very little research has been conducted concerning the examination of the consistency of rankings generated by the various interval representations, including center, lower, upper and radius intervals. The robustness and stability of interval-valued CODAS, EDAS and TOPSIS methods in respect of Flexible Manufacturing System selection problems have not been extensively investigated for a set of equal criteria and same data sets yet. As a result, there is no appropriate way for decision-makers to assess the reliability and consistency of various interval-valued MCDM techniques.

The aim of this paper is to design a robust interval-valued MCDM framework for Flexible Manufacturing System selection in an environment of uncertainty. The offered approach includes center-based, lower-bound, upper-bound and radius-based interval representations along with interval-valued CODAS, EDAS and TOPSIS methods to compare alternative FMS systems. Entropy-based weighting method is used for determining criterion weights and interval arithmetic is applied for uncertain decision data. The FMS selection problem related to the manufacture of tractor parts is selected as an example to verify the proposed methodology. Comparative analysis is then conducted to examine ranking consistency, stability, and robustness in different intervals and different decision-making approaches.

The principal contributions of this research are summarized as follows:

(i) A unified interval-valued MCDM framework is proposed for Flexible Manufacturing System selection under uncertainty.
(ii) Four interval representations, namely centre, lower, upper, and radius interval models, are incorporated within a common decision-making framework.
(iii) Interval-valued CODAS, EDAS, and TOPSIS methods are comparatively evaluated using identical decision data.
(iv) Entropy-based objective weighting is integrated to determine criterion importance without subjective bias.
(v) A comparative effectiveness framework based on average ranking, ranking variance, and consistency measures is developed to assess the robustness and stability of different interval-valued MCDM methods.
(vi) The effectiveness of the proposed framework is demonstrated through a realistic manufacturing decision problem.

The rest of this paper is structured as follows. In Section 2, we summarize the available literature about interval-valued multi-criteria decision-making methods and FMS selection. Section 3 gives the general framework of the proposed methodology. Section 4 covers the preliminary discussion of some concepts like interval arithmetic, interval order relations, entropy weight technique, and interval aggregation operators. Section 5 outlines the interval-valued ranking procedure and effectiveness analysis. Section 6 includes the numerical example for the illustration of the proposed framework. Finally, Section 7 gives the application of the proposed framework for the FMS selection problem along with the ranking results obtained.

2. Literature Review

Multi-Criteria Decision Making (MCDM) methods have developed alongside the necessity to tackle uncertainty, vagueness, and conflicting evaluation criteria in engineering and manufacturing. One of the first and most significant contributions to the field can be attributed to Bellman and Zadeh [1], who were among the first researchers to apply fuzzy sets in the domain of decision-making. Their seminal paper offered a formalized approach for dealing with objectives and constraints in terms of fuzzy sets, thus offering decision-makers an improved way to model the vagueness and uncertainty involved in their decisions. Based on the idea of fuzzy aggregation, Yager [2] developed the Ordered Weighted Averaging (OWA) operator, which marked a breakthrough in the field of multi-criteria aggregation approaches. As opposed to classical weighted averaging, the OWA operator is capable of controlling the behavior of aggregation depending on the decision attitude of the decision maker, which can be either optimistic or pessimistic. Due to this ability of controlling decision-making behaviors, OWA operators prove to be very useful for solving engineering decision-making problems in which the criteria are of different importance and uncertainty levels. Later, Fodor et al. (1995) [3] studied mathematical properties of OWA operators and formulated the theory behind OWA operators. Later work involving OWA aggregation was carried out by Cho (1995) [4], who developed a fuzzy aggregation scheme for modular neural networks based on OWA operators. Although initially applied to neural networks, the technique showed that OWA operators could be used to aggregate data from various uncertain sources, which is a feature very useful in assessing manufacturing systems. More recently, Yager and Filev (1999) [5] extended the traditional OWA paradigm by developing the Induced Ordered Weighted Averaging (IOWA) operator. While the standard OWA approach uses the actual values of data as a criterion for their ordering, the IOWA operator employs an external induction function for determining the order. As decision-making problems became progressively more complicated, it was evident that precise numeric models were no longer adequate. In order to deal with the problem, Pan et al. (2000) [6] suggested what may be among the first interval valued MADM models for resource planning strategy. With their model, Pan et al. (2000) considered evaluation information in the form of interval numbers rather than numeric data; thus, providing a realistic way to describe uncertainty in decision-making environment. Their interval model took into account variance in experts' evaluations as well as measurement errors. Further research was directed at developing aggregation operators that would be able to deal with interval-valued data. In their paper Xu and Da [7] developed an Operator called Ordered Weighted Geometric (OWG). This operator extended the concept of OWA by using multiplicative aggregation. In contrast to additive aggregation, the OWG operator is especially useful for decision-making problems in which criteria have proportional relationships with each other. It has been found to perform better in cases when there are substantial interactions among criteria. Acknowledging the importance of contextual information regarding the significance of evaluation criteria, Yager [8] further advanced the theory of aggregation by presenting a class of induced aggregation operators that decouple the process of ranking criteria with their actual values. This approach made the model more flexible by enabling decision makers to rank alternatives based on external factors rather than just numeric values. This feature is particularly useful in the case of manufacturing systems evaluation, where expert opinions play an important role in making decisions. Along with advancements in the area of aggregation theory, Hansen and Walster [9] underlined the need to use the concept of interval analysis as a mathematical means to account for uncertainty. They showed the existence of computationally efficient algorithms for the operation of intervals as well as proved that interval models give reliable bounds for uncertain numbers regardless of probability assumptions. One of the major uses of fuzzy multi-criteria decision making in manufacturing has been introduced by Kulak and Kahraman [10]. They have compared fuzzy and crisp axiomatic design methodologies to evaluate flexible manufacturing systems (FMS). Their work clearly demonstrates the usefulness of the fuzzy approach as it improves decision quality due to the inclusion of expert judgments through the use of fuzzy modeling. It has also been shown in their work that the use of fuzzy decision-making tools provides much more realistic evaluations of alternatives than the crisp models. All these groundbreaking works laid down the theory of fuzzy decision making, aggregation operators, interval arithmetic, and interval-valued MCDM. It was shown that uncertainty could be properly handled by means of fuzzy sets and interval values, and flexible aggregation operators help to enhance the accuracy of multicriteria evaluations. These achievements became the foundation for many interval-valued decision-making approaches developed over the following years and considerably impacted research on Flexible Manufacturing Systems Selection in uncertain environment.

The emergence of interval valued MCDM methodologies became more apparent after extending classical decision-making methodologies to situations of uncertainty. Interval valued TOPSIS methodology was developed by Jahanshahloo et al. [11], which employs intervals for representing attribute ratings and then prioritizes the alternatives based on how close they are to positive and negative ideal solution intervals. In order to optimize under interval uncertainty, Mahato and Bhunia [12] employed interval arithmetic for global optimization problems and proposed reliable algorithms to deal with the uncertain parameters. Likewise, Karmakar et al. (2009) [13] adopted an approach of interval optimization by dividing the feasible domain into several subdomains. Considering the evaluation of Flexible Manufacturing System (FMS), Rao and Parnichkun [14] introduced a multi-criteria decision-making model through the use of combinatorial decision analysis technique that successfully prioritized FMS options. At about the same time, Sayadi et al. [15] extended VIKOR approach into the interval framework using the optimistic attitude of the decision-makers in order to achieve better compromises. In the same year, Jahanshahloo et al. [16] enhanced the interval TOPSIS approach by integrating the notion of interval efficiency. The behavioral elements associated with decision-making have been addressed by Chen et al. [17], where the elements of loss aversion and prospect theory have been embedded in interval-valued TODIM method. Beliakov & James (2011) [18] made the Induced Ordered Weighted Averaging (IOWA) operator more flexible by using context-dependent ordering of the criteria. Some of the MCDM methods which were designed from applications' point of view were also suggested in this period. These include the Euclidean Distance-Based Approach (EDBA), suggested by Rao and Singh in 2011 [19] for manufacturing systems selection, and Preference Selection Index (PSI) method suggested by Maniya and Bhatt in 2011 [20], which is capable of evaluating options without pairwise comparison. Tsaur in 2011 [21] developed TOPSIS using interval information and risk attitudes, while Jahanshahloo et al. in 2011 [22] proposed an extension of TOPSIS using DEA approach. These researches have greatly enhanced the power of interval-valued MCDM through the extension of classical approaches, behavior-oriented preferences, and robustness in decision-making, which is the basis of current interval-based decision support systems in manufacturing applications.

The combination of interval arithmetic with optimization and aggregation methods contributed even more to the application of MCDM approaches. Sahoo et al. [23] presented interval-valued multi-objective reliability optimization problems with the aid of genetic algorithms. The efficiency of the interval arithmetic approach in the field of engineering optimization was shown by Zeng et al. [24], who introduced the concept of Fuzzy Generalized Ordered Weighted Averaging with Distance operator, and Sayyadi and Makui [25], who extended the approach of ELECTRE III with interval numbers. Ahn and Choi [26] proposed a weighting procedure for ordinal scales based on the OWA approach. More developments in the area include the use of interval-valued MOORA and MULTIMOORA approaches in sustainable decision making by Kracka and Zavadskas [27]. Probabilistic weighted averaging distance operator was developed by Merigo [28], whereas importance weighted continuous generalized OWA operator was developed by Liu et al. [29] for the purpose of interval-valued group decision making. Karande and Chakraborty [30] successfully implemented the MACBETH approach for FMS selection, while Dymova et al. [31] proposed an interval-based TOPSIS approach. Moreover, research conducted by Stanujkic et al. [32], Merigo et al. [33], Emrouznejad and Marra (2014) [34], Leon et al. [35], Wang et al. [36], and Aggarwal [37] improved OWA operators, probabilistic weighting, cross-efficiency analysis, and compensatory decision models.

Some of the recent studies have been done to improve the effectiveness of IV-MCDM models. The authors of the EDAS method were Keshavarz Ghorabaee et al. [38], while CODAS method was developed by Keshavarz Ghorabaee et al. [39]. These methods showed high efficiency in evaluation and ranking of alternatives within the context of uncertainty. Gonzalez-Hidalgo et al. (2016) [40] and Garg (2016) [41] improved aggregation methods using OWA and IF operators. The authors of IVEDAS model were Stanujkic et al. [42]. Meanwhile, Hafezalkotob and Hafezalkotob (2017) [43,44] proposed the interval-valued MULTIMOORA and VIKOR methods.

Though many researchers have made progress in the field of interval-valued MCDM, research work in this field is mostly about the development or application of individual approaches. In fact, a comparative analysis of the interval-valued CODAS, EDAS, and TOPSIS in a single framework is yet to be made. Furthermore, the impact of various forms of intervals, namely center, lower bound, upper bound, and radius, on the consistency of ranking is also yet to be explored in detail. In light of the above facts, this study proposes a framework for interval-valued MCDM in FMS selection through a comparative analysis of the above mentioned three methods, along with an investigation into the impact of different interval types on decision making.

3. Methodology

The present research develops an interval-valued multi-criteria decision-making method for choosing the best FMS in an uncertain environment. Both performance ratings of alternative solutions and weights of criteria are assumed to be given in interval format. At first, an interval-valued decision matrix is formed based on expert assessments. Next, the obtained matrix is normalized and interval-valued weights are added to the evaluation process. In order to compare FMSs, three well-known interval MCDM techniques known as CODAS, EDAS, and TOPSIS are employed. To consider the influence of various types of intervals on the results, four interval formats such as center, lower, upper, and radius are examined. Rankings generated from the three techniques are analyzed to determine their consistency, stability, and reliability. The comparative analysis, which is conducted using the basis of rank concordance and average ranks, offers an effective model for determining the best FMS.

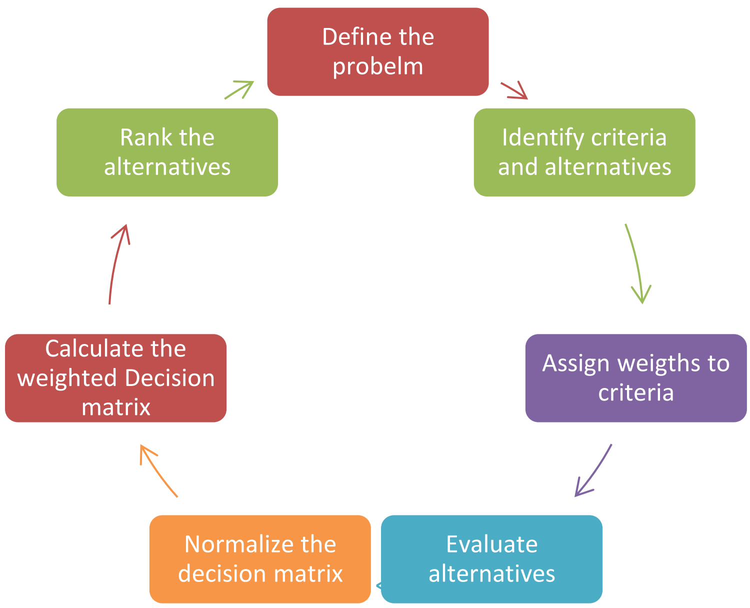

Fig. 1. MCDM problems steps using interval-valued criteria

4. Preliminaries

4.1 Interval mathematics and interval order relations [23]

An interval number A is closed interval represented as $A=[a_L,a_R]$, where $a_L,a_R \in \mathrm{R}$ are the lower and upper bounds, respectively. It can also be expressed in center-radius form as $A=<a_c,a_w>=\{x: a_c-a_w \le x \le a_c+a_w, x\in\Re\}$,

where $a_c=\dfrac{(a_R+a_L)}{2}$ and $a_w=\dfrac{(a_R-a_L)}{2}$ (1)

A real number a is a special case of an interval, represented by $a=[a,a]$, with zero radius.

Here, we shall give some basic formulas of Interval Mathematics as follows:

Let $A=[a_L,a_R]$ and $B=[b_L,b_R]$ be two intervals.

Addition: $A+B=[a_L,a_R]+[b_L,b_R]=[a_L+b_L,a_R+b_R]$ (2)

Subtraction: $A-B=[a_L,a_R]-[b_L,b_R]=[a_L-b_R,a_R-b_L]$ (3)

Scalar multiplication: $\alpha A=\alpha[a_L,a_R]=\begin{cases}[\alpha a_L,\alpha a_R] & \text{if } \alpha\ge 0\\ [\alpha a_R,\alpha a_L] & \text{if } \alpha<0\end{cases}$

Multiplication: $A\times B=[a_L,a_R]\times[b_L,b_R]=[\min(a_Lb_L,a_Lb_R,a_Rb_L,a_Rb_R),\max(a_Lb_L,a_Lb_R,a_Rb_L,a_Rb_R)]$

Division: $\dfrac{A}{B}=A\times\dfrac{1}{B}=[a_L,b_R]\times\left[\dfrac{1}{b_R},\dfrac{1}{b_L}\right]$, *provided* $0\notin[b_L,b_R]$

Exponential: $\exp(A)=[\exp(a_L),\exp(a_R)]$

Logarithm: $\log(A)=[\log(a_L),\log(a_R)]$

4.2 Integral power of an interval

According to Hansen and Walster [9], the integral power of an interval is defined by

$$A^n=[a_L,a_R]^n=\begin{cases}[1,1] & \text{if } n=0\\ [a_L^n,a_R^n] & \text{if } a_L\ge 0 \text{ or if } n \text{ is odd}\\ [a_R^n,a_L^n] & \text{if } a_R\le 0,\ n \text{ is even}\\ [0,\max(a_L^n,a_R^n)] & \text{if } a_L\le 0\le a_R,\ n>0 \text{ is even}\end{cases} \quad (4)$$

4.3 n^th root of an interval

Karmakar, Mahato, and Bhunia (2009) [13] defined the n^th root of an interval as follows:

$$\sqrt[n]{A}=\sqrt[n]{[a_L,a_R]}=\begin{cases}[\sqrt[n]{a_L},\sqrt[n]{a_R}] & \text{if } a_L\ge 0 \text{ or if } n \text{ is odd}\\ [0,\sqrt[n]{a_R}] & \text{if } a_L\le 0,\ a_R\ge 0 \text{ and } n \text{ is even}\\ \varphi & \text{if } a_R<0 \text{ and, } n \text{ is even}\end{cases} \quad (5)$$

Where φ is the empty interval and this definition is called n^th root of an interval $A=[a_L,a_R]$. The rational power of an interval $A=[a_L,a_R]$ is defined as follows:

$$(A)^{\frac{p}{q}}=(A)^{\frac{1}{q}} \text{ or equivalently, } (A)^{\frac{p}{q}}=\exp\left\{\frac{p}{q}\log A\right\}$$

4.4 Interval power of an interval

Let $A=[a_L,a_R]$ and $B=[b_L,b_R]$ be two intervals, then

$$(A)^B=[a_L,a_R]^{[b_L,b_R]}=\begin{cases}[e^{\min(b_L\log a_L,b_L\log a_R,b_R\log a_L,b_R\log a_R)}, \\ e^{\max(b_L\log a_L,b_L\log a_R,b_R\log a_L,b_R\log a_R)}] & \text{if } a_L\geq 0 \\ \text{a complex interval} & \text{if } a_L<0\end{cases} \tag{6}$$

$$\exp(B)=[e,e]\lfloor b_L,b_R\rfloor$$
$$=[e^{\min(b_L\log e,b_L\log e,b_R\log e,b_R\log e)},$$
$$e^{\max(b_L\log e,b_L\log e,b_R\log e,b_R\log e)}]$$
$$=[e^{b_L},e^{b_R}]=[\exp(b_L),\exp(b_R)]$$

4.5 Order relation of interval numbers

The criteria of our MCDM problem would be interval valued. So, to find the optimal alternatives of the said problem, the order relations of interval numbers play an important role in decision making process. For this purpose, we have considered the order relations proposed by Sahoo et.al [23]. Let $A=[a_L,a_R]$ and $B=[b_L,b_R]$ be two intervals. Then these two intervals may be of intervals of the following three types:

Type-1: Two intervals are disjoint.
Type-2: Two intervals are partially overlapping. (7)
Type-3: One of the intervals contained the other one.

It is to be noted that both the intervals $A=[a_L,a_R]$ and $B=[b_L,b_R]$ will be equal in case of fully overlapping intervals. That is $A=B$ if $a_L=b_L$ and $a_R=b_R$.

Definition 1. The order $A=[a_L,a_R]=\prec a_c,a_w\succ$ and $B=[b_L,b_R]=\prec b_c,b_w\succ$, then

$$\begin{aligned}&(i)\, A>_{\max} B\Leftrightarrow a_c>b_c \text{ for Type-1 and Type-2 intervals}\\&(ii)\, A>_{\max} B\Leftrightarrow \text{either } a_c\geq b_c \wedge a_w<b_w \text{ or}\\&a_c\geq b_c \wedge a_L<b_L \text{ for Type-3 intervals}\end{aligned} \tag{8}$$

Clearly the order relation $>_{\max}$ is reflexive and transitive but not symmetric.

Definition 2. The order relation $<_{\min}$ intervals $A=[a_L,a_R]=\prec a_c,a_w\succ$ and $B=[b_L,b_R]=\prec b_c,b_w\succ$, then

$$\begin{aligned}&(i)\, A<_{\min} B\Leftrightarrow a_c<b_c \text{ for Type-1 and Type-2 intervals}\\&(ii)\, A<_{\min} B\Leftrightarrow \text{either } a_c\leq b_c \wedge a_w<b_w \text{ or}\\&a_c\leq b_c \wedge a_L<b_L \text{ for Type-3 intervals}\end{aligned} \tag{9}$$

Clearly the order relation $<_{\min}$ is reflexive and transitive but not symmetric.

4.6 Entropy Weight Method

Entropy was coined by R. Clausius in thermodynamics (1865) and was subsequently extended by C. E. Shannon to information theory in 1948. Entropy is the measure of uncertainty or disorder. In MCDM, the entropy weight approach derives the weights of criteria objectively according to the information content in the decision matrix, whereby the more variant criteria have higher weightage.

Let $Z = \left(Z_{ij}\right)_{m\times n}$ be the decision matrix with m alternatives $A_i(i=1,2,.........,m)$ and n criterion $A_j(j=1,2,.........,n)$. The entropy weights are computed as follows:

Step 1: Compute $P_{ij} = \dfrac{z_{ij}}{\sum_{i=1}^{m} z_{ij}}$ (10)

Step 2: Compute $E_j = -\dfrac{1}{\log(m)}\sum_{i=1}^{m} P_{ij}\log(P_{ij})$ (11)

It is to be that $P_{ij}\log P_{ij} \to 0,\ when\, P_{ij} \to 0$

Step 3: Compute $U_j = 1 - E_j$ (12)

Step 4: Compute $w_j = \dfrac{U_j}{\sum_{j=1}^{n} U_j} = \dfrac{1-E_j}{\sum_{j=1}^{n}(1-E_j)}$ (13)

Definition 3 Let $R_j,\ j=1,2,...,n$ be the collection of real numbers; then weighted averaging aggregation operator can be defined as follows:
$WA_w(R_1,R_2,...,R_n) = w_1R_1 + w_2R_2 + ... + w_nR_n$, where $w_1 = (w_1,w_2,...,w_n)^T$ is the weighted vector of $R_j,\ j=1,2,...,n$ such that $0 \le w_j \le 1$, $j=1,2,...,n$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 4 Let $R_j,\ j=1,2,...,n$ be the collection of real numbers; then weighted order averaging aggregation operator can be defined as follows:
$WOA_w(R_1,R_2,...,R_n) = w_1R_{\sigma(1)} + w_2R_{\sigma(2)} + ... + w_nR_{\sigma(n)}$, where $w_1 = (w_1,w_2,...,w_n)^T$ is the weighted vector of $R_j,\ j=1,2,...,n$ such that $0 \le w_j \le 1$, $j=1,2,...,n$ and $\sum_{j=1}^{n} w_j = 1$. Here $(\sigma_{(1)},\sigma_{(2)},...,\sigma_{(n)})$ is a permutation of $(1,2,...,n)$ such that $R_{\sigma(j)} < R_{\sigma(j-1)}$, for all $j=1,2,...,n$.

Definition 5 Let $A_j = [a_{jL}, a_{jR}],\ j=1,2,...,n$ be the collection of real valued intervals numbers; then weighted averaging aggregation operator of interval valued numbers can be defined as follows:
$IWA_w(A_1,A_2,...,A_n) = w_1[a_{1L},a_{1R}] + w_2[a_{2L},a_{2R}] + ... + w_n[a_{nL},a_{nR}]$, where $w_1 = (w_1,w_2,...,w_n)^T$ is the weighted vector of $R_j,\ j=1,2,...,n$ such that $0 \le w_j \le 1$, $j=1,2,...,n$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 6 Let $A_j = [a_{jL}, a_{jR}],\ j=1,2,...,n$ be the collection of real valued interval numbers; then weighted order averaging aggregation operator of interval valued numbers can be defined as follows:
$IWOA_w(A_1,A_2,...,A_n) = w_1[a_{\sigma(1)L},a_{\sigma(1)R}] + w_2[a_{\sigma(2)L},a_{\sigma(2)R}] + ... + w_n[a_{\sigma(n)L},a_{\sigma(n)R}]$,where $w_1 = (w_1,w_2,...,w_n)^T$ is the weighted vector of $R_j,\ j=1,2,...,n$ such that $0 \le w_j \le 1$, $j=1,2,...,n$ and $\sum_{j=1}^{n} w_j = 1$. Here $(\sigma_{(1)},\sigma_{(2)},...,\sigma_{(n)})$ is a permutation of $(1,2,...,n)$ such that $A_{\sigma(j)} <_{\min} A_{\sigma(j-1)}$, for all $j=1,2,...,n$.

5. Proposed Algorithm

5.1 Proposed Ranking Algorithm

The proposed algorithm determines the ranking of interval-valued alternatives using entropy-based weighting and interval order relations.

Step 1: Construct the interval-valued decision matrix consisting of m alternatives and n evaluation criteria.

Step 2: Construct the normalized decision matrix by applying the appropriate normalization technique to non-beneficial criteria while retaining the normalized values for beneficial criteria. Subsequently, determine the interval order relations by considering the following cases:

(i) **Type 1:** Two intervals are disjoint.
(ii) **Type 2:** Two intervals are partially overlapping.
(iii) **Type 3:** One interval is completely contained within the other.

Step 3: Construct the centre interval matrix.
Step 4: Construct the radius interval matrix.
Step 5: Construct the lower interval decision matrix.
Step 6: Construct the upper interval decision matrix.
Step 7: Compute the entropy weights for the centre, radius, lower, and upper interval matrices by:

(a) Normalizing the interval values.
(b) Computing the entropy values.
(c) Determining the degree of diversification.
(d) Calculating the entropy weights.
(e) Repeating Steps (a)–(d) for all interval matrices.

Step 8: Determine the ranking of the alternatives by:

(a) Multiplying the normalized interval values by their corresponding entropy weights.
(b) Aggregating the weighted values of each alternative to obtain the interval scores.
(c) Determining the corresponding lower, upper, centre, and radius values of the interval scores.

- (d) Assigning Rank 1 to the alternative with the highest interval score and ranking the remaining alternatives in descending order.

Step 9: Construct the reordered interval decision matrix according to the obtained ranking.
Step 10: Repeat Steps 7–9 until the rankings of all alternatives are obtained, resulting in a complete ordering of the alternatives.

5.2 Comparative Effectiveness table

The comparative effectiveness table is generated by calculating the average rank of each alternative rank across every method after the individual rankings of each FMS alternative have been determined using the present MCDM methods. Additionally, the following formulas are used to evaluate each alternative's stability and consistency.

Step 1: The average rank of an alternative A_i' over n MCDM method is computed using

$$R_i' = avg_{A_i'} = \frac{1}{n}\sum_{j=1}^{n} R_{ij}'$$

where $R_{ij}' =$ Rank of alternative A_i' in method j.

Step 2: Compute the variance of ranks of each alternative to measure stability using

$\text{var}_{A_i'} = \frac{1}{n}\sum_{j=1}^{n}(R_{ij}' - R_i')^2$ where lower variance indicates higher stability.

Step 3: Compute consistency levels based on range using

$range_{A_i'} = \max\left(R_{ij}'\right) - \min\left(R_{ij}'\right)$

If $range_{A_i'} = 0$ then "Very high"

Else if $range_{A_i'} \le 1$ then "High"

Else if $range_{A_i'} \le 2$ then "Moderate"

Else "Low"

End if

Step 4: Compute consistency levels based on standard deviation using

$\partial_{A_i'} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(R_{ij}' - R_i')^2}$

If $\partial_{A_i'} < 0.2$ then "Very high"

Else if $0.2 \le \partial_{A_i'} < 0.5$ then "High"

Else if $0.5 \le \partial_{A_i'} < 1$ then "Moderate"

Else if $\partial_{A_i'} \ge 1$ then "Low"

Else "Unstable"

6. Numerical example

For illustrating the efficiency of the developed interval MCDM model, the case study of FMS selection problem by Kulak and Kahraman [10] is used here. In this case study, the optimal FMS has to be selected from four alternatives considering six criteria, which are Quality of Result (QR), Ease of Use (EA), Competitiveness (CO), Adaptability (AD) and Expandability (EX) as benefit criteria, while Annual Depreciation and Maintenance (ADM) as cost criterion. The interval valued decision matrix is shown in Table 1. In contrast to earlier researches that have used interval values after converting them into crisp form, the current study uses interval valued decision matrix as it retains the uncertainty of the process of evaluation. Interval valued CODAS, EDAS, and TOPSIS methods are used here to rank FMS options using four different intervals, i.e., center interval, lower interval, upper interval, and radius interval. These ranking results are compared to results given in the literature for consistency, robustness, and effectiveness of the approach adopted. The methodology adopted is presented step by step in Figure 1.

Table 1
Interval-Valued Decision Matrix for Flexible Manufacturing System (FMS) Selection

	ADM(X$1000)	QR	EA	CO	AD	EX
FMS-1	[210,240]	[18,20]	[13,20]	[16,20]	[12,18]	[12,16]
FMS-2	[80,120]	[12,17]	[9,14]	[12,17]	[15,17]	[14,18]
FMS-3	[180,220]	[8,12]	[10,14]	[13,18]	[19,20]	[9,14]
FMS-4	[140,170]	[7,10]	[8,14]	[13,17]	[12,16]	[11,13]

7. Solving an MCDM Problem and Analysis

The interval-valued MCDM model is then utilized for the FMS selection problem based on the interval decision matrix given in Table 1. As ADM is a cost criterion, it will be converted into a benefit criterion using the proposed normalization method, while the other benefit criteria will remain as they are. Table 2 displays the resulting normalized interval decision matrix.

Table 2
Normalized Interval Decision Matrix

	ADM(X$1000)	**QR**	**EA**	**CO**	**AD**	**EX**
FMS-1	[0,30]	[18,20]	[13,20]	[16,20]	[12,18]	[12,16]
FMS-2	[120,160]	[12,17]	[9,14]	[12,17]	[15,17]	[14,18]
FMS-3	[20,60]	[8,12]	[10,14]	[13,18]	[19,20]	[9,14]
FMS-4	[70,100]	[7,10]	[8,14]	[13,17]	[12,16]	[11,13]

The center interval matrix is computed using Eq. (1) by calculating the midpoint of each interval. The resulting matrix is presented in Table 3.

Table 3
Center Interval Matrix

	ADM(X$1000)	**QR**	**EA**	**CO**	**AD**	**EX**
FMS-1	15.00	19.00	16.50	18.00	15.00	14.00
FMS-2	140.00	14.50	11.50	14.50	16.00	16.00
FMS-3	40.00	10.00	12.00	15.50	19.50	11.50
FMS-4	85.00	8.50	11.00	15.00	14.00	12.00

Using the entropy weighting method, the criterion weights are obtained from the center interval matrix. The corresponding ranking results are presented in Table 4.

Table 4
Ranking Based on Center Interval Matrix

Alternative	**[Lower value, Upper value]**	**Interval Range**	**Rank**
FMS-1	[13.41,19.53]	16.47	2
FMS-2	[18.79,25.24]	22.02	1
FMS-3	[11.90,17.97]	14.94	4
FMS-4	[13.59,18.97]	16.28	3

The radius interval matrix is obtained from Eq. (1) by calculating half the width of each interval. The computed values are listed in Table 5.

Table 5
Radius Interval Matrix

	ADM(X$1000)	QR	EA	CO	AD	EX
FMS-1	15.00	1.00	3.50	2.00	3.00	2.00
FMS-2	20.00	2.50	2.50	2.50	1.00	2.00
FMS-3	20.00	2.00	2.00	2.50	0.50	2.50
FMS-4	15.00	1.50	3.00	2.00	2.00	1.00

Entropy weights are computed using the radius matrix, and the resulting rankings are shown in Table 6.

Table 6
Ranking Based on Radius Interval Matrix

Alternative	[Lower value, Upper value]	Interval Range	Rank
FMS-1	[13.29,19.45]	16.37	3
FMS-2	[19.77,26.22]	22.99	1
FMS-3	[12.56,18.64]	15.60	4
FMS-4	[14.21,19.60]	16.91	2

The lower interval matrix is formed using the lower bounds of all interval values, as shown in Table 7.

Table 7
Lower Interval Matrix

	ADM(X$1000)	**QR**	**EA**	**CO**	**AD**	**EX**
FMS-1	0.00	18.00	13.00	16.00	12.00	12.00
FMS-2	120.00	12.00	9.00	12.00	15.00	14.00
FMS-3	20.00	8.00	10.00	13.00	19.00	9.00
FMS-4	70.00	7.00	8.00	13.00	12.00	11.00

The entropy weighting method is applied to the lower interval matrix, and the ranking results are summarized in Table 8.

Table 8
Ranking Based on Lower Interval Matrix

Alternative	[Lower value, Upper value]	Interval Range	Rank
FMS-1	[11.66,20.82]	16.24	4
FMS-2	[31.67,42.46]	37.07	1
FMS-3	[12.77,23.22]	17.99	3
FMS-4	[20.72,29.29]	25.01	2

Similarly, the upper interval matrix is obtained by considering the upper bounds of the interval data. The resulting matrix is presented in Table 9.

Table 9
Upper Interval Matrix

	ADM(X$1000)	**QR**	**EA**	**CO**	**AD**	**EX**
FMS-1	30.00	20.00	20.00	20.00	18.00	16.00
FMS-2	160.00	17.00	14.00	17.00	17.00	18.00
FMS-3	60.00	12.00	14.00	18.00	20.00	14.00
FMS-4	100.00	10.00	14.00	17.00	16.00	13.00

The entropy weights calculated from the upper interval matrix produce the rankings shown in Table 10.

Table 10
Ranking Based on Upper Interval Matrix

Alternative	[Lower value, Upper value]	Interval Range	Rank
FMS-1	[13.70,19.21]	16.46	2
FMS-2	[16.46,22.08]	19.27	1
FMS-3	[11.79,17.02]	14.41	4
FMS-4	[12.30,17.07]	14.69	3

To investigate the influence of interval ordering, the interval attributes are rearranged according to the proposed interval ordering strategy considering Type-I (disjoint), Type-II (partially overlapping), and Type-III (contained intervals). The reordered interval matrix is presented in Table 11. The interval values from Table 11 are ordered based on the new ordering approach of intervals that classifies the relationship between two intervals into three types, namely Type-I (disjoint intervals), Type-II (partial overlap intervals), and Type-III (one interval included in the other). The above ordering keeps the uncertainty information intact and allows a basis for comparing two intervals. The interval-valued MCDM method using the above ordering approach of intervals is then carried out using center, radius, lower, and upper interval representation. The ranking outcomes are tabulated in Tables 12–15.

Table 11
Ordered Interval Matrix

	ADM(X$1000)	**QR**	**EA**	**CO**	**AD**	**EX**
FMS-1	[18,20]	[16,20]	[13,20]	[12,16]	[12,18]	[0,30]
FMS-2	[120,160]	[15,17]	[14,18]	[12,17]	[12,17]	[9,14]
FMS-3	[20,60]	[19,20]	[13,18]	[10,14]	[9,14]	[8,12]
FMS-4	[70,100]	[13,17]	[12,16]	[11,13]	[8,14]	[7,10]

Table 12
Ranking Based on Ordered Center Interval Matrix

Alternative	**[Lower value, Upper value]**	**Interval Range**	**Rank**
FMS-1	[11.12,21.08]	16.10	3
FMS-2	[19.99,26.72]	23.35	1
FMS-3	[12.49,18.78]	15.63	4
FMS-4	[14.34,20.10]	17.22	2

Table 13
Ranking Based on Ordered Radius Interval Matrix

Alternative	**[Lower value, Upper value]**	**Interval Range**	**Rank**
FMS-1	[11.01,21.04]	16.02	3
FMS-2	[18.93,25.19]	22.06	1
FMS-3	[12.56,18.46]	15.51	4
FMS-4	[13.89,19.25]	16.57	2

Table 14
Ranking Based on Ordered Lower Interval Matrix

Alternative	**[Lower value, Upper value]**	**Interval Range**	**Rank**
FMS-1	[12.00,20.90]	16.45	4
FMS-2	[31.90,42.43]	37.16	1
FMS-3	[13.65,23.88]	18.76	3
FMS-4	[21.16,29.63]	25.39	2

Table 15
Ranking Based on Ordered Upper Interval Matrix

Alternative	[Lower value, Upper value]	Interval Range	Rank
FMS-1	[10.92,21.28]	16.10	2
FMS-2	[16.87,22.46]	19.66	1
FMS-3	[12.49,17.68]	15.08	4
FMS-4	[12.76,17.63]	15.19	3

Finally, the proposed approach is compared with several well-known MCDM methods reported in the literature.

Table 16
Comparison of Rankings Obtained Using Existing MCDM Methods

Ranking for FMS various MCDM Method	FMS-1	FMS-2	FMS-3	FMS-4
EDBA (Rao & Singh, 2011) [19]	2	1	3	4
PSI method (Maniya & Bhatt, 2011) [20]	2	1	3	4
MACBETH (Karande & Chakraborty, 2013) [30]	2	1	4	3
Interval valued TOPSIS (Mathew & Thomas, 2019) [46]	2	1	4	3
Interval valued EDAS (Mathew & Thomas, 2019) [46]	2	1	4	3
Interval valued CODAS (Mathew & Thomas, 2019) [46]	2	1	4	3

Table 17
Rankings Obtained by the Proposed Interval-Valued MCDM Framework

Proposed MCDM Method Rank	FMS-1	FMS-2	FMS-3	FMS-4
Center weight with interval matrix	2	1	4	3
Radius weight with interval matrix	3	1	4	2
Lower weight with interval matrix	4	1	3	2
Upper weight with interval matrix	2	1	4	3
Center weight with order interval matrix	3	1	4	2
Radius weight with order interval matrix	3	1	4	2
Lower weight with order interval matrix	4	1	3	2
Upper weight with order interval matrix	2	1	4	3

Table 17 presents the ranks obtained via the proposed IV-MCDM methodology. It is evident from the rankings that FMS-2 always ranks first regardless of the type of interval representation used and the ordering technique applied to intervals. In other words, FMS-2 is superior to the other three FMSs being considered for adoption. There are slight differences in the ranking of FMS-1, FMS-3, and FMS-4, which can be attributed to the use of different interval representations (center, radius, lower, and upper bounds) and ordering of intervals. However, this does not affect the ranking order much since the results remain consistent. The ranking order shown in Table 17 is consistent with those of other MCDM methodologies available in Table 16.

7. Results and Comparative Analysis

The interval-valued MCDM approach is analyzed using eight ranking methods, each using four intervals (center, radius, lower, and upper) and two kinds of matrices (interval and ordered interval). For validation of the suggested method, the results were compared with six known MCDM approaches: EDBA [19], PSI [20], MACBETH [30], and interval-valued TOPSIS, EDAS, and CODAS [46].

The results of the comparison are shown in Tables 16 and 17 as well as Figures 2 and 3. It is clear from the analysis that FMS-2 always secures the first rank using all MCDM methods available. Small variations can be found in FMS-1, FMS-3, and FMS-4, which are caused by their respective interval and weights systems. However, the ranking patterns are very similar and reliable, which shows the reliability of the proposed system. Further investigation into the stability of these alternatives was conducted by calculation of various statistics, such as average rank, variance, standard deviation, and rank range. The comparison of effectiveness of these alternatives is provided in Table 18 below, and the graph is presented in Figure 4.

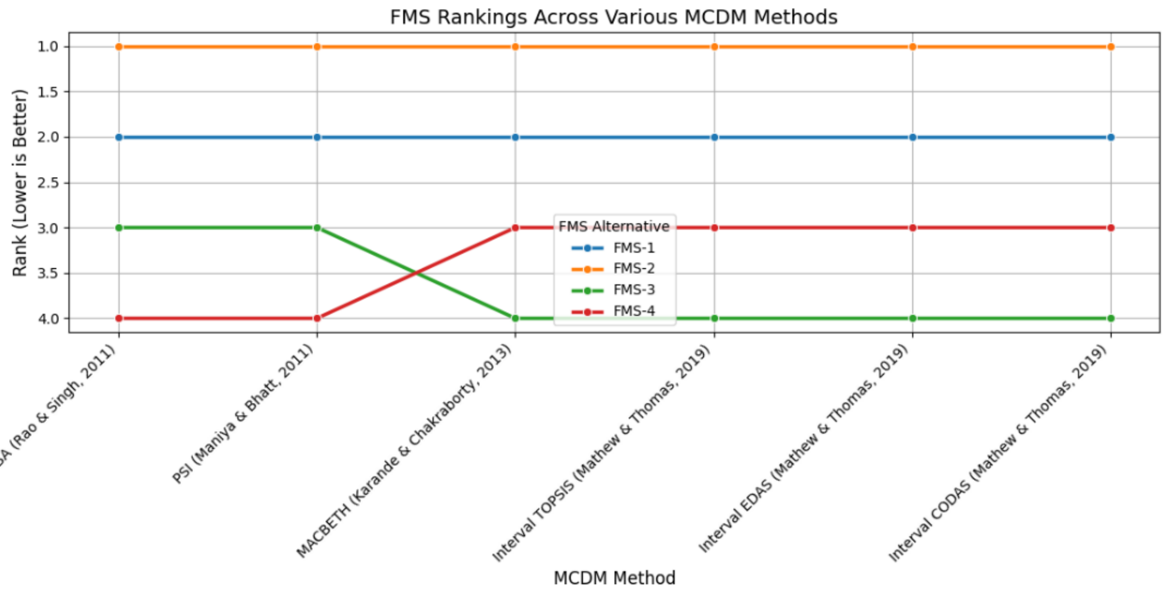

Fig. 2. Comparison of Rankings Obtained Using Existing MCDM Methods.

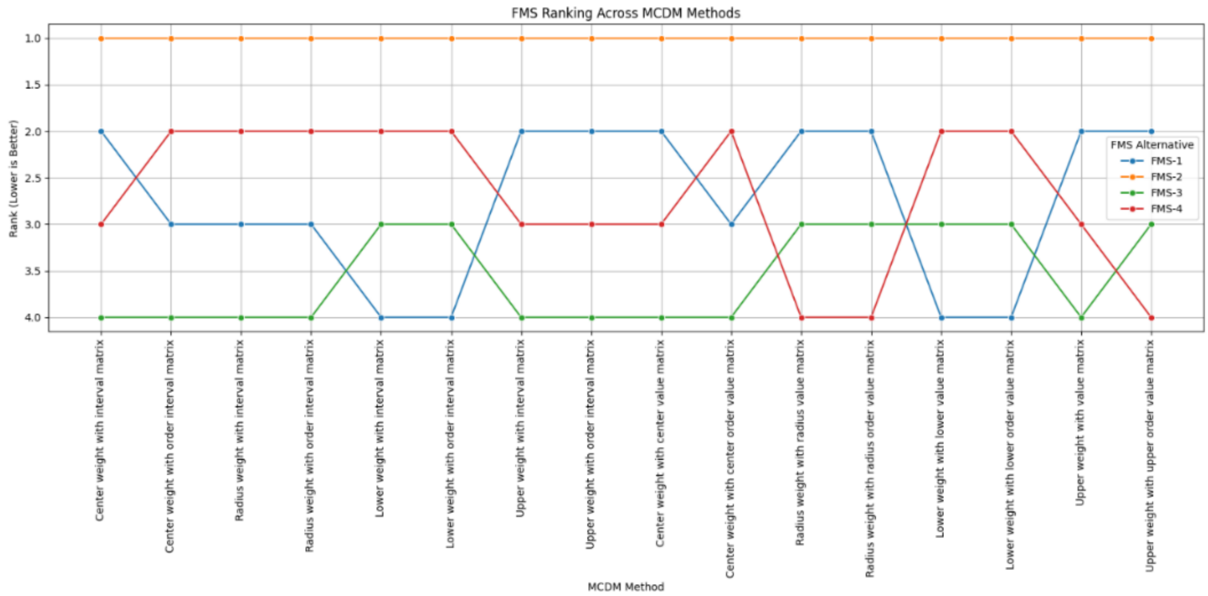

Fig. 3. Rankings Obtained Using the Proposed Interval-Valued MCDM Framework.

Table 18

Comparative Effectiveness Analysis of FMS Alternatives

Alternative	Average Rank	Variance	Standard Deviation	Range	Stability	Consistency
FMS-1	2.75	0.73	0.86	2	Low	Moderate
FMS-2	1.00	0.00	0.00	0	High	Very High
FMS-3	3.56	0.26	0.51	1	Medium	High
FMS-4	2.69	0.63	0.79	2	Low	Moderate

Based on Table 18, the most stable and consistent ranking is produced by FMS-2, where the average rank of the alternative equals 1.00, its variance is 0.00, its standard deviation is 0.00, and its rank range is 0, which means that all the rankings are identical. The worst-performing solution is FMS-3, whose rankings are fairly consistent, while FMS-1 and FMS-4 get moderate rankings with fair consistency. The findings prove that the suggested interval-valued MCDM approach produces reliable, stable and consistent rankings.

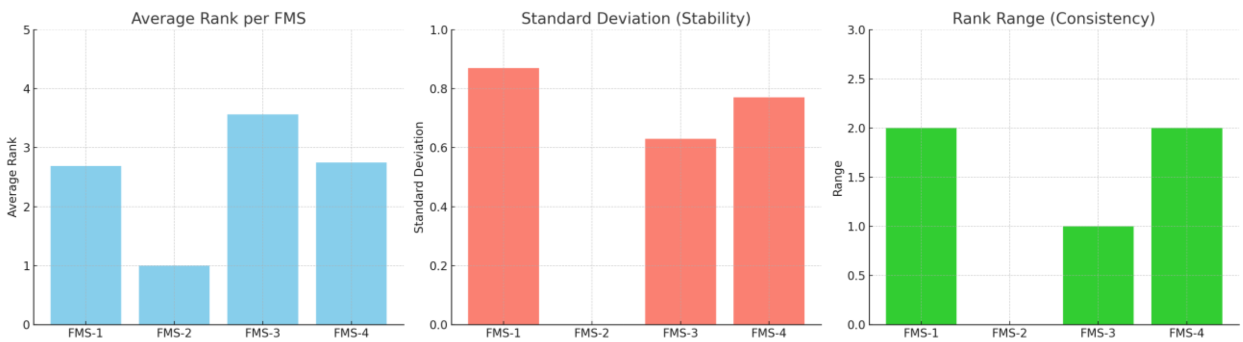

Fig. 4. Comparative Analysis of the Stability and Consistency of FMS Alternatives.

8. Conclusion and Future Directions

The research presents a robust MCDM model based on interval-valued criteria for the selection of the best FMS in uncertain decision-making environment. The application of CODAS, EDAS, and TOPSIS methods in combination with center, radius, lower, and upper intervals makes it possible to capture uncertainty in data and give a consistent ranking of the considered alternatives. The comparison analysis revealed that the proposed approach provides stable results that are comparable to those obtained with traditional MCDM models and show high consistency, with FMS-2 being recognized as the best choice regardless of the situation. The results proved the effectiveness of the proposed model in supporting decision making in manufacturing processes in case of uncertain information. Further studies may be aimed at the development of this model for application in fuzzy environments, such as Circular Intuitionistic Fuzzy [53], Fermatean picture [54], Decomposed Pythagorean [55] and Spherical Fuzzy Hypersoft sets [56], in addition to machine learning and time-series analysis.

Author Contributions

Conceptualization, S.L.G. and L.S.; methodology, S.L.G. and L.S.; software, S.L.G.; validation, S.L.G. and L.S.; formal analysis, S.L.G.; investigation, S.L.G.; resources, L.S.; data curation, S.L.G.; writing—original draft preparation, S.L.G.; writing—review and editing, S.L.G. and L.S.; visualization, S.L.G.; supervision, L.S.; All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

The datasets generated during and/or analyzed during the current study are included in the manuscript.

Conflicts of Interest

The authors declare that there is no known competing interest directly or indirectly influencing the writing of this paper.

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References

[1] Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science*, *17*(4), B-141. https://doi.org/10.1287/mnsc.17.4.B141

[2] Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multicriteria decision making. *IEEE Transactions on systems, Man, and Cybernetics*, *18*(1), 183-190. 10.1109/21.87068

[3] Fodor, J., Marichal, J. L., & Roubens, M. (1995). Characterization of the ordered weighted averaging operators. *IEEE Transactions on Fuzzy Systems*, *3*(2), 236-240. 10.1109/91.388176

[4] Cho, S. B. (1995). Fuzzy aggregation of modular neural networks with ordered weighted averaging operators. *International journal of approximate reasoning*, *13*(4), 359-375.

[5] Yager, R. R., & Filev, D. P. (1999). Induced ordered weighted averaging operators. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, *29*(2), 141-150. 10.1109/3477.752789

[6] Pan, J., Teklu, Y., Rahman, S., & de Castro, A. (2000). An interval-based MADM approach to the identification of candidate alternatives in strategic resource planning. *IEEE transactions on power systems*, *15*(4), 1441-1446. 10.1109/59.898125

[7] Xu, Z. S., & Da, Q. L. (2002). The ordered weighted geometric averaging operators. *International Journal of Intelligent Systems*, *17*(7), 709-716. 10.1002/int.10045

[8] Yager, R. R. (2003). Induced aggregation operators. *Fuzzy sets and systems*, *137*(1), 59-69. https://doi.org/10.1016/S0165-0114(02)00432-3

[9] Hansen, E., & Walster, G. W. (Eds.). (2003). *Global optimization using interval analysis: revised and expanded* (Vol.264). CRC press.

[10] Kulak, O., & Kahraman, C. (2005). Multi-attribute comparison of advanced manufacturing systems using fuzzy vs. crisp axiomatic design approach. *International Journal of Production Economics*, *95*(3), 415-424. 10.1016/j.ijpe.2004.02.009

[11] Jahanshahloo, G. R., Lotfi, F. H., & Izadikhah, M. (2006). An algorithmic method to extend TOPSIS for decision-making problems with interval data. *Applied mathematics and computation*, *175*(2), 1375-1384. 10.1016/j.amc.2005.08.048

[12] Mahato, S. K., & Bhunia, A. K. (2006). Interval-arithmetic-oriented interval computing technique for global optimization. *Applied Mathematics Research Express*, *2006*, 69642. 10.1155/AMRX/2006/69642

[13] Karmakar, S., Mahato, S. K., & Bhunia, A. K. (2009). Interval oriented multi-section techniques for global optimization. *Journal of computational and Applied Mathematics*, *224*(2), 476-491. 10.1016/j.cam.2008.05.025

[14] Rao, R. V., & Parnichkun, M. (2009). Flexible manufacturing system selection using a combinatorial mathematics-based decision-making method. *International Journal of Production Research*, *47*(24), 6981-6998. 10.1080/00207540802389227

[15] Sayadi, M. K., Heydari, M., & Shahanaghi, K. (2009). Extension of VIKOR method for decision making problem with interval numbers. *Applied mathematical modelling*, *33*(5), 2257-2262. 10.1016/j.apm.2008.06.002

[16] Jahanshahloo, G. R., Lotfi, F. H., & Davoodi, A. R. (2009). Extension of TOPSIS for decision-making problems with interval data: Interval efficiency. *Mathematical and computer modelling*, *49*(5-6), 1137-1142. 10.1016/j.mcm.2008.07.009

[17] Chen, F. D., Zhang, X., Kang, F., Fan, Z. P., & Chen, X. (2010, August). A method for interval multiple attribute decision making with loss aversion. In *2010 International Conference of Information Science and Management Engineering* (Vol. 1, pp. 453-456). IEEE. 10.1109/ISME.2010.155

[18] Beliakov, G., & James, S. (2011). Induced ordered weighted averaging operators. In *Recent developments in the ordered weighted averaging operators: Theory and practice* (pp. 29-47). Berlin, Heidelberg: Springer Berlin Heidelberg. 10.1007/978-3-642-17910-5

[19] Rao, R. V., & Singh, D. (2011). Evaluating flexible manufacturing systems using Euclidean distance-based integrated approach, International Journal of Decision Sciences, Risk and Management, 3(1-2), 32-53. https://doi.org/10.1504/IJDSRM.2011.040746

[20] Maniya, K. D., & Bhatt, M. G. (2011). The selection of flexible manufacturing system using preference selection index method. *International Journal of Industrial and Systems Engineering*, *9*(3), 330-349. https://doi.org/10.1504/IJISE.2011.043142

[21] Tsaur, R. C. (2011). Decision risk analysis for an interval TOPSIS method. *Applied Mathematics and Computation*, *218*(8), 4295-4304. 10.1016/j.amc.2011.10.001

[22] Jahanshahloo, G. R., Khodabakhshi, M., Lotfi, F. H., & Goudarzi, M. M. (2011). A cross-efficiency model based on super-efficiency for ranking units through the TOPSIS approach and its extension to the interval case. *Mathematical and Computer Modelling*, *53*(9-10), 1946-1955. 10.1016/j.mcm.2011.06.045

[23] Sahoo, L., Bhunia, A. K., & Kapur, P. K. (2012). Genetic algorithm based multi-objective reliability optimization in interval environment. *Computers & Industrial Engineering*, *62*(1), 152-160. https://doi.org/10.1016/J.CIE.2011.09.003

[24] Zeng, S., Su, W., & Le, A. (2012). Fuzzy generalized ordered weighted averaging distance operator and its application to decision making. *International Journal of Fuzzy Systems*, *14*(3), 402-412.

[25] Sayyadi, M., & Makui, A. (2012). A new view to uncertainty in Electre III method by introducing interval numbers. *Decision Science Letters*, *1*(1), 33-38. 10.5267/j.dsl.2012.06.002

[26] Ahn, B. S., & Choi, S. H. (2012). Aggregation of ordinal data using ordered weighted averaging operator weights. *Annals of Operations Research*, *201*, 1-16. 10.1007/s10479-012-1169-3

[27] Kracka, M., & Zavadskas, E. K. (2013). Panel building refurbishment elements effective selection by applying multiple-criteria methods. *International Journal of Strategic Property Management*, *17*(2), 210-219. 10.3846/1648715X.2013.808283

[28] Merigo, J. M. (2013). The probabilistic weighted averaging distance and its application in group decision making. *Kybernetes*, *42*(5), 686-697. 10.1108/K-06-2013-0107

[29] Liu, H., Cai, J., & Martínez, L. (2013). The importance weighted continuous generalized ordered weighted averaging operator and its application to group decision making. *Knowledge-Based Systems*, *48*, 24-36.

[30] Karande, P., & Chakraborty, S. (2013). Evaluation and selection of flexible manufacturing systems using MACBETH method. *International Journal of Services and Operations Management*, *16*(1), 123-144. https://doi.org/10.1504/IJSOM.2013.055576

[31] Dymova, L., Sevastjanov, P., & Tikhonenko, A. (2013). A direct interval extension of TOPSIS method. *Expert Systems with Applications*, *40*(12), 4841-4847. 10.1016/j.eswa.2013.02.022

[32] Stanujkic, D., Magdalinovic, N., Milanovic, D., Magdalinovic, S., & Popovic, G. (2014). An efficient and simple multiple criteria-model for a grinding circuit selection based on MOORA method. *Informatica*, *25*(1), 73-93. 10.15388/Informatica.2014.05

[33] Merigó, J. M., Casanovas, M., & Yang, J. B. (2014). Group decision making with expertons and uncertain generalized probabilistic weighted aggregation operators. *European Journal of Operational Research*, *235*(1), 215-224. 10.1016/j.ejor.2013.10.011

[34] Emrouznejad, A., & Marra, M. (2014). Ordered weighted averaging operators 1988–2014: A citation-based literature survey. *International Journal of Intelligent Systems*, *29*(11), 994-1014. 10.1002/int.21673

[35] León, T., Ramón, N., Ruiz, J. L., & Sirvent, I. (2014). Using induced ordered weighted averaging (iowa) operators for aggregation in cross-efficiency evaluations. *International Journal of Intelligent Systems*, *29*(12), 1100-1116. 10.1002/int.21685

[36] Wang, H., Mo, H., Sadiq, R., Hu, Y., & Deng, Y. (2015). Ordered visibility graph weighted averaging aggregation operator: A methodology based on network analysis. *Computers & Industrial Engineering*, *88*, 181-190. https://doi.org/10.1016/j.cie.2015.06.021

[37] Aggarwal, M. (2015). Generalized compensative weighted averaging aggregation operators. *Computers & Industrial Engineering*, *87*, 81-90. https://doi.org/10.1016/j.cie.2015.04.021

[38] Keshavarz Ghorabaee, M., Zavadskas, E. K., Olfat, L., & Turskis, Z. (2015). Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS). *Informatica*, *26*(3), 435-451. 10.15388/Informatica.2015.57

[39] Ghorabaee, M. K., Zavadskas, E. K., Turskis, Z., & Antucheviciene, J. (2016). A new combinative distance-based assessment (CODAS) method for multi-criteria decision-making. *Economic Computation and Economic Cybernetics Studies and Research*, *50*(3), 25-44.

[40] González-Hidalgo, M., Massanet, S., Mir, A., & Ruiz-Aguilera, D. (2016, July). Edge image aggregation method using ordered weighted averaging functions. In *2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)* (pp. 1355-1362). 10.1109/FUZZ-IEEE.2016.7737847

[41] Garg, H. (2016). Some series of intuitionistic fuzzy interactive averaging aggregation operators. *SpringerPlus*, *5*(1), 999. 10.1186/s40064-016-2591-9

[42] Stanujkic, D., Zavadskas, E. K., Ghorabaee, M. K., & Turskis, Z. (2017). An extension of the EDAS method based on the use of interval grey numbers. *Studies in Informatics and Control*, *26*(1), 5-12. 10.24846/v26i1y201701

[43] Hafezalkotob, A., & Hafezalkotob, A. (2017). Interval MULTIMOORA method with target values of attributes based on interval distance and preference degree: biomaterials selection. *Journal of Industrial Engineering International*, *13*(2), 181-198. 10.1007/s40092-016-0176-4

[44] Hafezalkotob, A., & Hafezalkotob, A. (2017). Interval target-based VIKOR method supported on interval distance and preference degree for machine selection. *Engineering Applications of Artificial Intelligence*, *57*, 184-196.https://doi.org/10.1016/j.engappai.2016.10.018

[45] Mathew, M., & Sahu, S. (2018). Comparison of new multi-criteria decision making methods for material handling equipment selection. *Management Science Letters*, *8*(3), 139-150. 10.5267/j.msl.2018.1.004

[46] Mathew, M., & Thomas, J. (2019). Interval valued multi criteria decision making methods for the selection of flexible manufacturing system. *International Journal of Data and Network Science*, *3*(4), 349-358.10.5267/j.ijdns.2019.4.001

[47] Kumar, K., & Chen, S. M. (2022). Multiple attribute group decision making based on advanced linguistic intuitionistic fuzzy weighted averaging aggregation operator of linguistic intuitionistic fuzzy numbers. *Information Sciences*, *587*, 813-824. https://doi.org/10.1016/j.ins.2023.119228

[48] Sen, S., Sahoo, L., Tiwary, K., Simic, V., & Senapati, T. (2023). Wireless sensor network lifetime extension via K-Medoids and MCDM techniques in uncertain environment. *Applied Sciences*, *13*(5), 3196. https://doi.org/10.3390/app13053196

[49] Guo, Z., Liu, J., Liu, X., Meng, Z., Pu, M., Wu, H., ... & Chen, F. (2024). An integrated MCDM model with enhanced decision support in transport safety using machine learning optimization. *Knowledge-Based Systems*, *301*, 112286. https://doi.org/10.1016/j.knosys.2024.112286

[50] Camgoz Akdag, H., Menekse, A., & Sahin, F. (2024). Cervical cancer risk assessment using machine learning integrated fuzzy MCDM methodology. *Journal of Intelligent & Fuzzy Systems*, *46*(2), 4573-4592. https://doi.org/10.3233/JIFS-23464

[51] Nasiri Khiavi, A., Tavoosi, M., Kamari Yekdangi, F., Sadikhani, M., & Kuriqi, A. (2024). Conjunct applicability of MCDM-based machine learning algorithms in mapping the sediment formation potential. *Environment, Development and Sustainability*, 1-31. 10.1007/s10668-024-05285-y

[52] Sahoo, L., & Ghosh, S. L. (2026). A Reinforcement Learning-Based Framework for Multi-Criteria Decision-Making. *Intelligent Systems Research and Applications Journal*, *2*, 129-145. https://doi.org/10.59543/ydm6xr12

[53] Khan, M. R., Raza, A., Ullah, K., & Seikh, M. R. (2026). Bridging Art and Logic: Evolving Artificial Intelligence for Emotionally Intelligent Art Communication Using Integrated PROMETHEE – Complex Circular Intuitionistic Fuzzy Dombi Framework. *Intelligent Systems Research and Applications Journal*, *2*, 305-339. https://doi.org/10.59543/hc7nh857

[54] Ali, U., Li, J., Senapati, T., Edalatpanah, S. A., Duleba, S., & Moslem, S. (2026). A hybrid decision-making model in a complex Fermatean picture fuzzy environment with application to emergency management. *Complex & Intelligent Systems*, *12*(1), 15.

[55] Turgay, S., Torkul, B., Aydin, A., & Özyurt, F. (2026). A Novel Decomposed Pythagorean Fuzzy CODAS Framework for Precision Customer Segmentation in Complex Market Environments. *Intelligent Systems Research and Applications Journal*, *2*, 160-198. https://doi.org/10.59543/r8xsq450

[56] Gul, Z., Hussain, A., Senapati, T., & Moslem, S. (2026). Performance of AI-Based Healthcare Projects with Decision Algorithm and Spherical Fuzzy Hypersoft Environments. *Informatics in Medicine Unlocked*, 101762.