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Decisions Under Deep Uncertainty: A Pythagorean Fuzzy Z-Number Framework for Urban Flood Risk Assessment

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ABSTRACT

Ambiguities are always present in real-world decision-making situations; in the case of floodplain urban growth, the planning process is further complicated by environmental unpredictability, limited data, and experts' subjective opinions. The current study proposes an enhanced multi-attribute decision-making (MADM) framework that incorporates Hamacher aggregation operations and Pythagorean fuzzy Z-numbers (PFZNS) to address the aforementioned issues. The proposed approach recommends constructing and investigating Hamacher geometric aggregation operators for decision-making problems involving multiple criteria. A variety of options are considered, depending on population density, infrastructure costs, land-use suitability, flood risk, and environmental effects, to evaluate flood risk in urbanized floodplains in Bangladesh using the model. The suggested method is implemented and tested using a case study in an urban floodplain in Bangladesh. The results show that the method yields consistent, stable rankings of alternatives, accounts for uncertainty in experts' opinions and flood information, and improves decision-making quality. In conclusion, the PFZN-MADM model presented in this study is reliable and flexible, enabling more confident decision-making in flood-prone areas to encourage sustainable city development.

1. Introduction

Decision-making (DM) is crucial for innovation, risk management, and organizational flexibility in solving problems, achieving goals, and using resources efficiently [1]. In real-world data, ambiguity is common, so Zadeh [2] proposed Fuzzy sets (FS), which are represented by a Membership Degree (MDg). To enhance the uncertainty modeling, Atanassov [3] introduced non-membership degrees (NMDg) and extended FS to intuitionistic fuzzy sets (IFS). But IFS restricts the sum of MDg and NMDg to be at most 1.

To overcome this limitation, Yager [4] introduced Pythagorean fuzzy sets (PyFS), which have more flexibility due to the use of the squared sum of the membership and non-membership degrees. To provide a more sophisticated perspective on uncertain judgments, Zadeh [5] introduced Z-numbers,

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which account for both restriction and reliability. Z-numbers can be used to make more accurate computer decisions and to represent human judgment [6]. Linear programming and function construction using basic mathematical operations for Z-numbers (addition, subtraction, multiplication, division, max, min, and square root) reduce uncertainty in MADM problem-solving [7].

Besides, the advanced aggregation operators (AOs) of PyFS have been shown in the literature. For the PyFS-based DM problems, Zhang and Xu [9] applied the TOPSIS method, and the averaging and geometric AOs were obtained by Yager and Abbasov [8]. Z-valuations, Hamacher-based aggregation operators, Einstein-based Pythagorean fuzzy aggregation, and other optimization techniques were used in subsequent research to expand the field [10]. These advancements enabled PFZN to improve its capacity to provide comprehensive multi-attribute decision-making solutions that address reliability and uncertainty. Figure 1 depicts the fundamental structure of a PFZN. Reliability and non-reliability information are represented by reliability and non-reliability values, respectively, while membership and non-membership information are represented by membership and non-membership values, respectively.

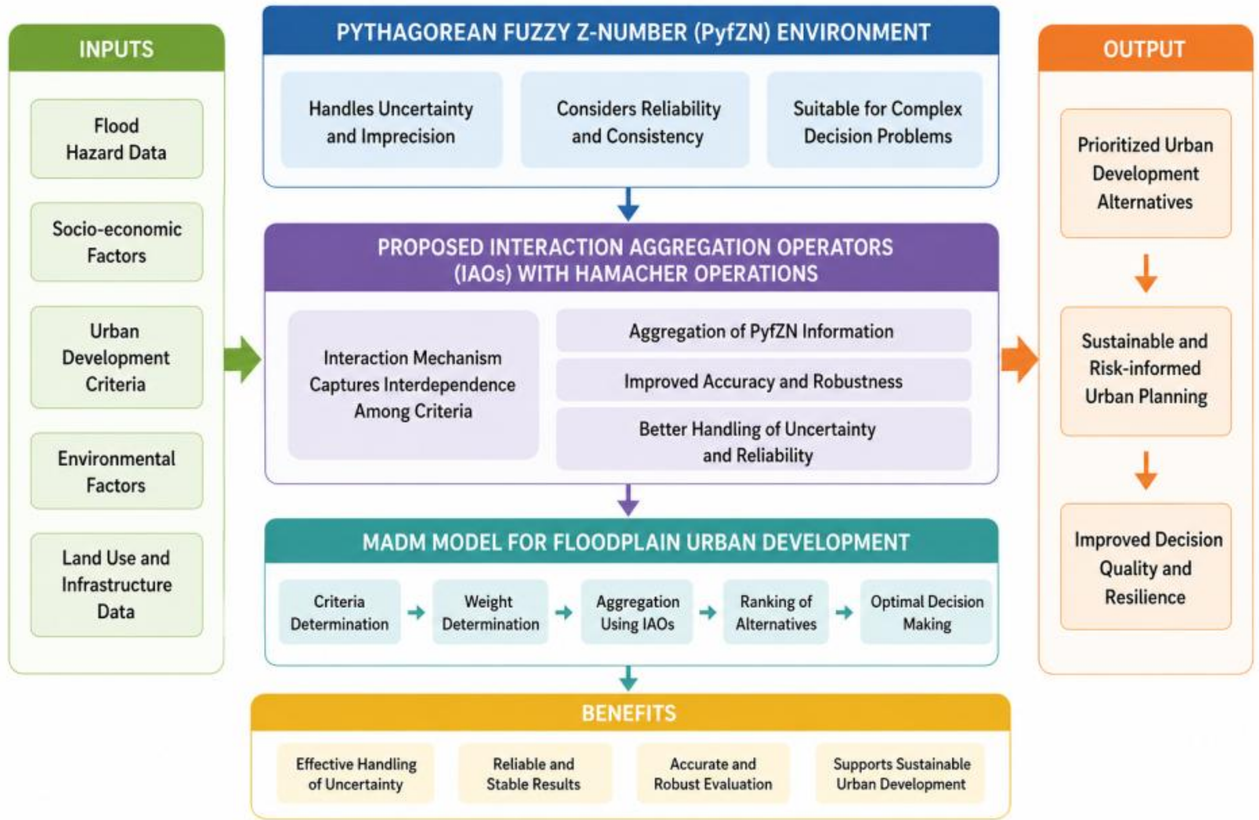

Fig. 1 Pythagorean fuzzy Z number system

To improve AOs in FS frameworks, researchers have explored a range of strategies. While t-norms (t-Nr) and t-conorms (t-CNr) offer variable algebraic aggregation, conventional AOs rely on straightforward algebraic rules. Variants like the Frank and Einstein norms have been widely used. Normalized AOs based on Bonferroni mean operators were proposed by Tešić and Marinković [11]. Power aggregation operators based on Hamacher t-norm and t-conorm for complicated intuitionistic fuzzy information and their use in decision-making issues were covered by Dong [12]. Huang [15] built AOs based on Hamacher t-norms, Ali et al. [13] developed Einstein aggregation operators, and Zeb et al. [14] introduced Hamacher aggregation operators for MADM. Deb et al. [17] extended the

AOs based on the Hamacher t-norms, while Yager [16] provided Z-valuations using Zadeh's Z-numbers. Numerous researchers used various decision-making techniques in [18, 19, 20, 21, 22].

New weighted averaging and geometric AOs tailored for PFZNs are proposed based on these results. When addressing preferences expressed by PFZNs, operators use Hamacher norm operations to provide a more comprehensive approach to handling ambiguity and uncertainty. The weights of PFZNs significantly impact alternative rankings; choosing weights carefully is essential. By providing updated tools that effectively address fuzzy uncertainty in multi-criteria evaluations, this research advances decision-making methodologies.

High population density, infrastructure costs, and flood risk are obstacles to the development of urban floodplains [23]. Incomplete urban planning data and ambiguous flood hazard data impede decision-making [24]. Economic, environmental, and social aspects are among the many factors that need to be considered [25]. IAOs include several requirements for robust urban planning, whereas PFZNs model data uncertainty and expert reliability [26, 27, 28]. This methodology helps decision-makers allocate resources and plan for mitigation by prioritizing development projects at risk of flooding.

1.1 Literature review and research gap

Despite their widespread use in decision-making processes, fuzzy sets (FS) and Pythagorean fuzzy Z-numbers (PFZNs) still have several drawbacks that limit their applicability to complex, uncertain real-world problems, such as the assessment of floodplain urban development in Bangladesh. To the best of the authors' knowledge, no prior study has prioritized urban development in floodplains by fully integrating PyfZNs with Hamacher-based interaction-aggregation operators. When making decisions about urban planning and flood risk assessment, current aggregation techniques often fail to capture both uncertainty and information accuracy simultaneously.

Additionally, there hasn't been much research comparing sophisticated aggregation operators with other multi-criteria decision-making (MCDM) methods used in floodplain development settings. In flood-prone areas such as Bangladesh, the application of PFZNs in urban development remains underexplored. Uncertainties related to population distribution, land-use variability, flood-hazard data, and shifting environmental circumstances are not adequately managed by current methods. Furthermore, only a small number of integrated decision-making frameworks support sustainable urban growth planning in flood-risk areas by accounting for technical, environmental, and social factors simultaneously. The goal of this effort is to create a new framework that integrates Hamacher interaction-aggregation operators with PFZN to close these gaps.

The suggested approach can be used to evaluate sustainable urban development in Bangladesh's floodplains and to make better decisions amid uncertainty.

1.2 Research goals

This study will apply PFZNs to develop new interaction-based aggregation operators to support the assessment of floodplain urban development under uncertainty. Furthermore, advanced Hamacher aggregation operators are proposed and implemented within the PFZN framework to enable a suitable assessment of urban development. Finally, a real-world case study of urban development in Bangladesh's floodplains will be used to test the proposed model and demonstrate its applicability to sustainable urban development in flood-prone areas. Overall, this study provides an integrated decision-making framework that strengthens the assessment and prioritization of FUD, supporting sustainable, risk-aware urban planning in Bangladesh amid uncertainty.

The following are the key contributions to the research. The following are the contributions of the research.

1.3 The main contributions

The key findings of this study are summarized below. To evaluate urban growth in the floodplain under uncertain conditions, novel operators, called interaction aggregation operators (IAOs), are first introduced within the framework of the Pythagorean fuzzy Z-number (PFZN). By combining PFZNs with interaction-based aggregation techniques to efficiently handle uncertainty, reliability, and complexity in multi-attribute decision-making, the proposed operators offer a theoretical breakthrough. Furthermore, a systematic algorithm, a flowchart, and a structured decision-making model are developed to facilitate the practical application of the recommended framework in urban planning.

A case study of floodplain urban development in a real-world setting in Bangladesh demonstrates the applicability of the proposed approach, which successfully prioritizes development alternatives. A comprehensive comparison shows that the proposed operators offer greater flexibility, accuracy, and robustness than existing aggregation methods, particularly in the presence of uncertainty and complex decision-making situations. All things considered, the suggested framework improves stability, reliability, and decision quality, offering a useful tool for risk-aware, sustainable floodplain urban development planning.

1.4 Research motivations

Classical fuzzy sets (FS), intuitionistic fuzzy sets (IFS), and Pythagorean fuzzy sets (PyFS) are generally insufficient to express such problems concurrently; they are ambiguous and unreliable in complex decision-making problems. Although PyFS is improved by the following modifications, it remains relatively simple. In this sense, it does not specifically offer a means to include levels of membership and non-membership. The reliability or accuracy of the available data. Although Z-numbers, which model uncertainty and reliability, have been introduced to address this problem, they remain difficult to use in multi-attribute decision-making frameworks.

The reliability-handling function of Z-numbers is another aspect of PyFS. The ability to openly model uncertainty with membership and non-membership is one of PFZN's main advantages for decision-makers. Additionally, they come with membership degrees, and the Z-number component ensures dependability. Expert opinions and the use of models are particularly significant in Bangladesh for floodplain urban development planning, as representation is essential because environmental and flood risk information is often inaccurate, incomplete, and ambiguous. Reliability and uncertainty are now combined, enabling more dependable, realistic decisions with PFZN. It facilitates data analysis under uncertainty, strategic urban planning, and simulations.

Figure 2 presents a concise flowchart summarizing the paper's structure, illustrating each section from the introduction to the conclusion and their corresponding purposes for clarity.

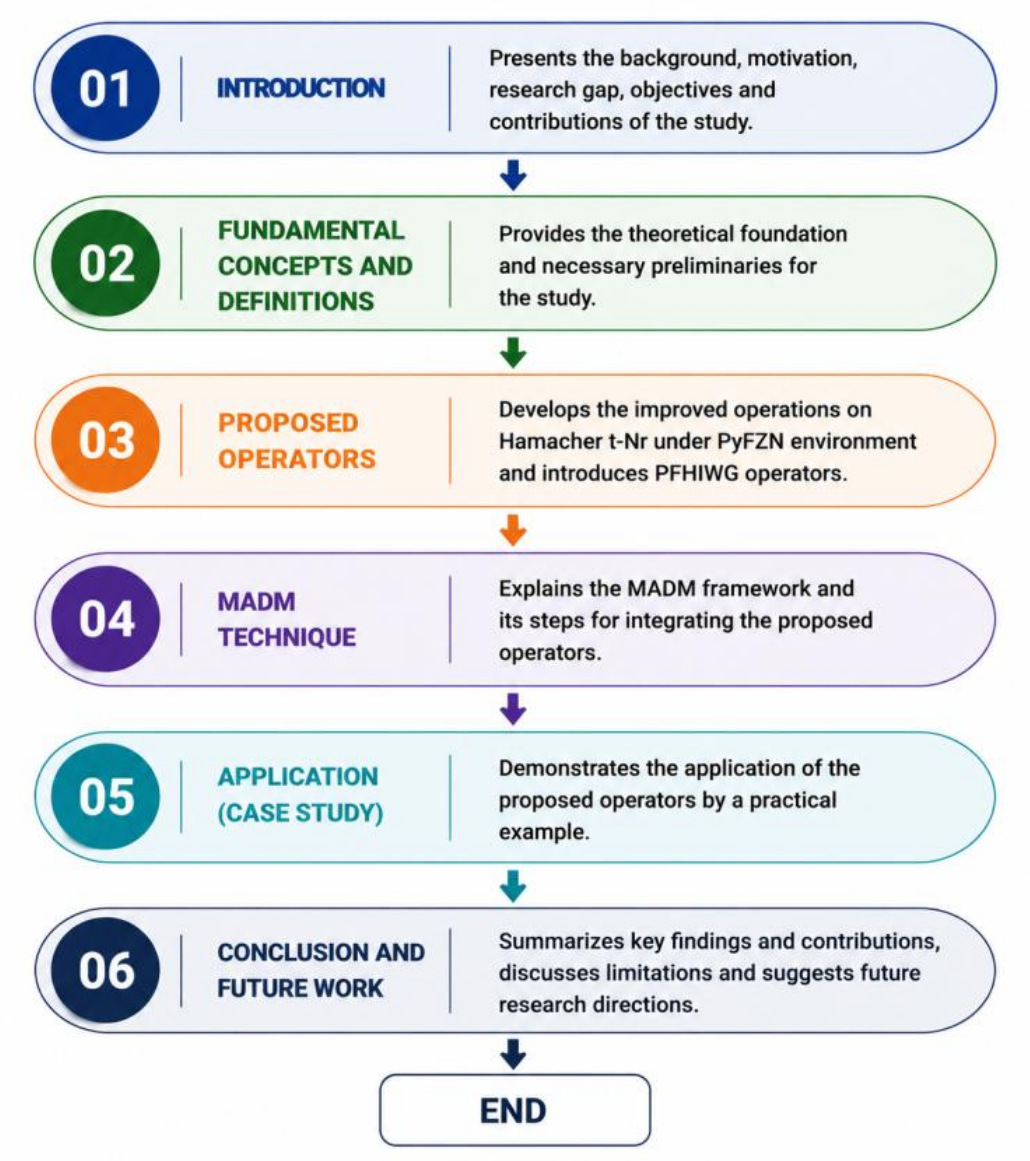

Fig. 2. A flowchart that describes the paper's layout

2. Preliminaries

This section will discuss basic definitions and rules to guide the construction of our paper.

Definition 2.1 [4]: A PyFS S defined in Y is an ordered pair presented by $S = \{\langle \mathrm{y}, \mathrm{m}(\mathrm{y}), w(\mathrm{y})\rangle | \mathrm{y} \in \mathrm{Y}\}$ where $\mathrm{m}(\mathrm{y}), w(\mathrm{y}): \mathrm{Y} \to [0,1]$ represent the MDg and NMDg of the element y such that $\mathrm{m}(\mathrm{y}), w(\mathrm{y}) \in [0,1]$ and $(\mathrm{m}(\mathrm{y}))^2 + (w(\mathrm{y}))^2 \leq 1$ for all y.

Definition 2.2 [5] A PFZN is an ordered pair $\omega = \{y, < (\mathrm{m}(y), w(y)), (n(y), l(y) > | y \in Y\}$ where $A = (\mathrm{m}, w)$ and $B = (n, l)$ are Pythagorean fuzzy numbers satisfying

$0 \leq \mathrm{m}(y)^2 + w(y)^2 \leq 1$ and $0 \leq n(y)^2 + l(y)^2 \leq 1$ for all y.

A represents the uncertain evaluation of a variable and B represents the reliability (confidence) of the evaluation.

Definition 2.3 [5] Let $\omega = \{y, < (\mathrm{m}(y), w(y)), (n(y), l(y) > | y \in Y\}$, then the score function (SF) for any PFZN is given as $SF(\omega) = \frac{1+\mathrm{mw}-\mathrm{nl}}{2} \in [0,1]$.

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3. Methodology

This section presents a DM method for solving MADM problems under the PFZN environment. In real-life situations, decision-making involves selecting the most suitable alternative from a set of available options. Decision-makers often rely on their knowledge, experience, and judgment; however, practical decision-making problems are usually complex and involve vague, uncertain, and imprecise information. Therefore, it is essential to represent such uncertainty accurately within a flexible mathematical framework.

This section presents the MCDM approach for PFZNs using the Hamacher aggregation operators.

Let $Q = \{Q_1, Q_2, Q_3, \dots, Q_p\}$ be a set of alternatives and $\mathrm{T} = \{\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3, \dots, \mathrm{T}_q\}$ be the set of criteria. Let $\breve{\mathrm{G}}$ be the WV such that $\sum_{j=1}^{q} \breve{\mathrm{G}}_j = 1,\ (j = 1, 2, \dots, n)$ where $\breve{\mathrm{G}}_j \in [0,1]$ and each $\breve{\mathrm{G}}_j$ represents the weight of T_j. The decision-maker (DM) reviews the alternatives for an attribute, and the assessment measures must be within the PFZNs. Let $\hat{\mathrm{W}} = (\hat{B}_{ij})_{p\times q}$ be the decision matrix that DM has supplied, where $\hat{B}_{ij}$ shows the PFZNs. The representation of $\hat{B}_{ij}$ is for alternatives Q related to criterion T_j.

In fact, an algorithm to fix a problem is being developed.

Step 1: In the form of PFZNs, the DM has expressed its personal viewpoint. Create a PFZN's decision matrix P by moving $\hat{\mathrm{W}} = (\hat{B}_{ij})_{p\times q}$ in the direction of the alternative Q with $\hat{B}_{ij} = \langle (\mathrm{m}_{ij}, w_{ij}), (n_{ij}, l_{ij}) \rangle$.

Step 2: The decision matrix should be normalized. If various criteria or features, such as cost ($\acute{\alpha}_c$) and benefit ($\acute{\alpha}_b$), are present. We handle each criterion or attribute uniformly by normalizing the decision matrix. If not, distinct criteria or traits ought to be combined in various ways.

$$\mathrm{T}_{ij} = \begin{cases} \hat{B}_{ij} & j \in \acute{\alpha}_b \\ \hat{B}_{ij}^{c} & j \in \acute{\alpha}_c \end{cases} \tag{23}$$

$\hat{B}_{ij}^{c}$ is the complement of $\hat{B}_{ij}$.

Step 3: The value of the alternative Q_i for different parameters T_j is determined using the PyFHWG operator for $\underline{\omega}_i = \langle (m_i, w_i), (n_i, l_i) \rangle$ $(i = 1,2, \dots, \mathrm{n})$ with $m_i, n_i \neq 1$ $(\mathrm{i} = 1,2, \dots, \mathrm{n})$ is defined

$$PyFHIWG(\underline{\omega}_1, \underline{\omega}_2 \ \dots, \underline{\omega}_n) =$$

$$\left\langle \begin{array}{c} \left(\frac{\mathfrak{Y}\oplus_{i=1}^{\dot{n}}(1-(\mathrm{w}_i)^2)^{\text{đ}_i} - \mathfrak{Y}\oplus_{i=1}^{\dot{n}}(1-(\mathrm{m}_i)^2-(w_i)^2)^{\text{đ}_i}}{\oplus_{i=1}^{\dot{n}}[1+(\mathfrak{Y}-1)(\mathrm{w}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{\dot{n}}(1-(\mathrm{w}_i)^2)^{\text{đ}_i}}, \frac{\oplus_{i=1}^{\dot{n}}(1+(\mathfrak{Y}-1)(\mathrm{w}_i)^2)^{\text{đ}_i} - \oplus_{i=1}^{\dot{n}}(1-(w_i)^2)^{\text{đ}_i}}{\oplus_{i=1}^{\dot{n}}[1+(\mathfrak{Y}-1)(\mathrm{w}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{\dot{n}}(1-(\mathrm{w}_i)^2)^{\text{đ}_i}} \right) \\ \left(\frac{\mathfrak{Y}\oplus_{i=1}^{\dot{n}}(1-(\mathrm{l}_i)^2)^{\text{đ}_i} - \mathfrak{Y}\oplus_{i=1}^{\dot{n}}(1-(\mathrm{n}_i)^2-(l_i)^2)^{\text{đ}_i}}{\oplus_{i=1}^{\dot{n}}[1+(\mathfrak{Y}-1)(\mathrm{l}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{\dot{n}}(1-(\mathrm{l}_i)^2)^{\text{đ}_i}}, \frac{\oplus_{i=1}^{\dot{n}}(1+(\mathfrak{Y}-1)(\mathrm{l}_i)^2)^{\text{đ}_i} - \oplus_{i=1}^{\dot{n}}(1-(l_i)^2)^{\text{đ}_i}}{\oplus_{i=1}^{\dot{n}}[1+(\mathfrak{Y}-1)(\mathrm{l}_i)^2]^{\text{đ}_i} - (\mathfrak{Y}-1)\oplus_{i=1}^{\dot{n}}(1-(\mathrm{l}_i)^2)^{\text{đ}_i}} \right) \end{array} \right\rangle.$$

Thus, from step 2 onward, the decision matrix is generated. This yields the collective value T_i for each alternative Q_i $(i = 1,2, \dots, p)$

Step 4: Utilizing Definition 2.3, determine the scoring functions for each T_i for PFZNs.

Step 5: To select the best option, rank each Q_i according to the score values.

The flow of the proposed MADM procedure based on PFZNs and Hamacher interactive geometric aggregation operators is illustrated in Figure 3.

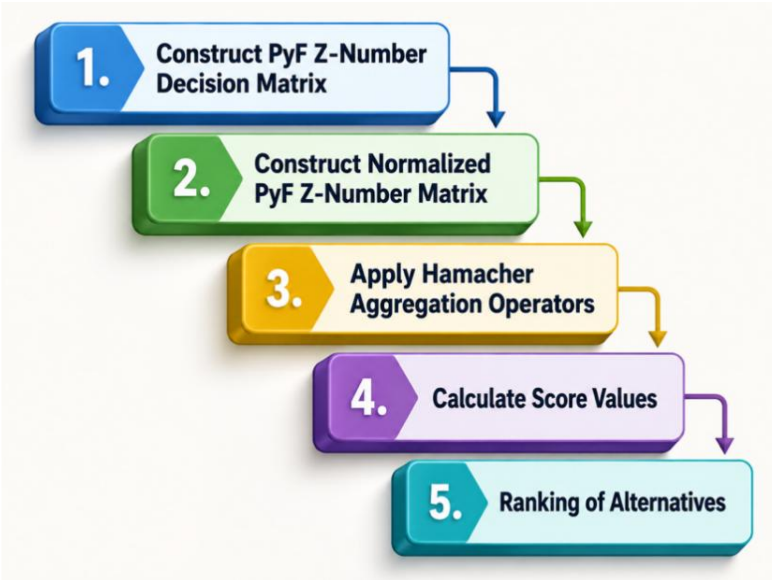

Fig. 3. Demonstration of **the** flowchart for the MADM technique

4. Practical example

This is a scenario of an enterprise in the industrial zone of the city of Juárez, Mexico, in the Maquiladoras sector, engaged in manufacturing and assembly. The company has taken on a project to reduce costs, and packaging has emerged as a potential area for savings. The proposed MADM technique for ranking supplier alternatives in the PFAZ environment is illustrated in Figure 4, which shows the step-by-step computation.

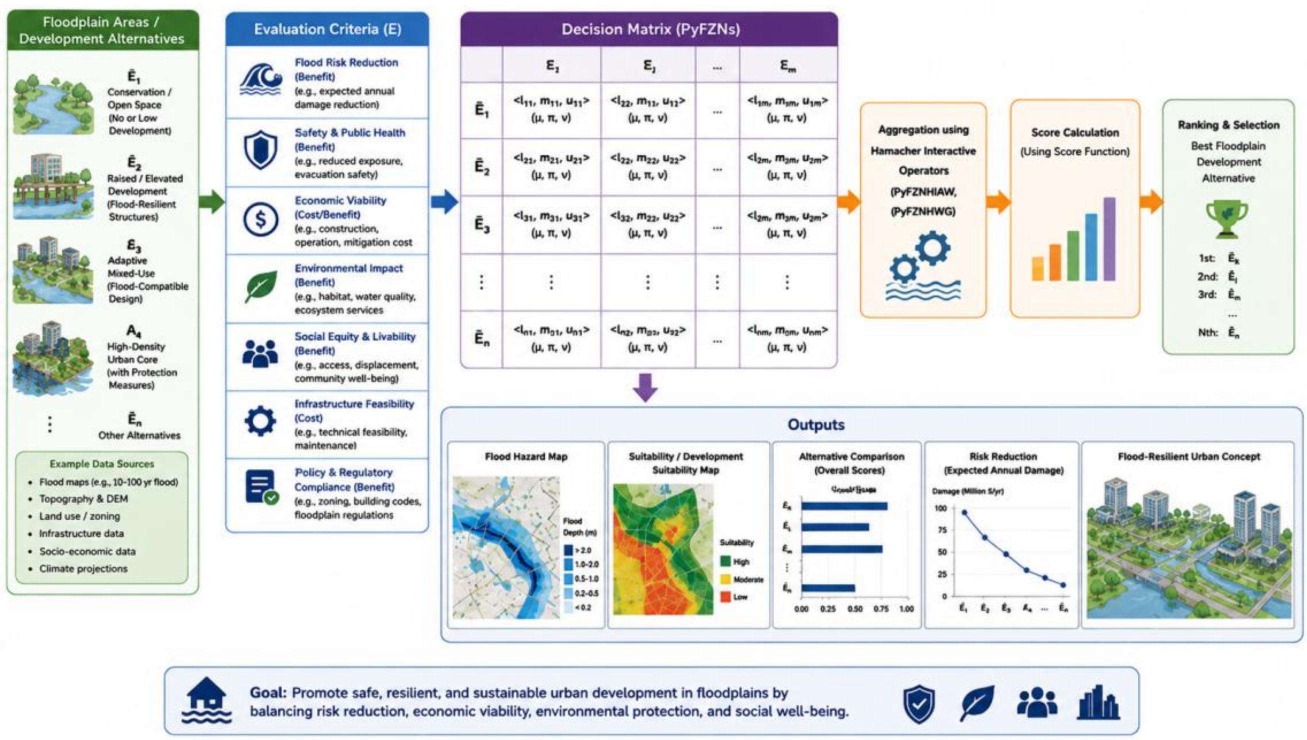

Fig. 4. Proposed alternatives ranking using MADM technique for Floodplain urban development

4.1 DECISION-MAKING CRITERIA FOR FLOODPLAIN URBAN DEVELOPMENT ASSESSMENT

The weighting of evaluation criteria was determined using PFZN based on expert judgments. Experts provided linguistic assessments regarding the importance of each criterion in floodplain urban development planning in Bangladesh. These linguistic terms were then converted into PFZN values to properly capture both uncertainty and reliability in expert opinions. The individual expert evaluations were aggregated and normalized so that the total weight of all criteria equals 1. The procedure guarantees that ambiguities in expert judgment and trustworthiness of information is taken into account in the decision-making process. The resulting derived criteria weights are more realistic, balanced, and reliable. For assessing urban development options in flood-prone areas. This study proposes a PFZN-based multi-attribute approach to prioritize floodplain urban development alternatives in Bangladesh. The aim of this approach to decision-making is to support sustainable urban planning amid flood risk and enhance decision quality amid uncertainty. The following criteria are used to assess and prioritize alternatives for urban development in floodplains according to the proposed PFZN-based MCDM framework.

The following criteria are considered for evaluating and ranking urban FLOODPLAIN alternatives under the proposed PFZN–Hamacher MCDM framework:

1. Flood Risk Exposure (T_1): The level of vulnerability of a region to flooding, including historical flood frequency, depth, and severity of flood events. Lower exposure indicates higher suitability for urban development.
2. Land Use and Suitability (T_2): The compatibility of land for urban expansion based on topography, soil type, drainage capacity, and existing land-use patterns.
3. Infrastructure Development Cost (T_3): The total cost required for constructing and maintaining infrastructure such as roads, drainage systems, utilities, and public facilities. Lower cost improves development feasibility.
4. Population Density and Distribution (T_4): The concentration and spatial distribution of population in floodplain areas, affecting demand for services and risk exposure.
5. Environmental Sustainability (T_5): The extent to which development preserves ecological balance, reduces environmental degradation, and supports green urban planning principles.
6. Disaster Resilience and Adaptability (T_6): The ability of urban systems to withstand, respond to, and recover from flood events and other climate-related disasters.
7. Infrastructure Accessibility (T_7): The availability and connectivity of transport networks, utilities, and public services that support urban development efficiency.
8. Socio-Economic Development Potential (T_8): The potential of an area to support economic growth, employment opportunities, and improved living standards.

4.2 UTILIZING THE PFZNIWG OPERATOR

Step 1. Enter the PF data keyed in by the decision maker in Table 4 to generate the decision matrix.

Table 4
PFZN's decision matrix taken by the decision maker, PFZNIWG Operator

	T_1	T_2	T_3	T_4
Q_1	⟨(0.60, 0.76), (0.72, 0.62)⟩	⟨(0.75, 0.47), (0.61, 0.51)⟩	⟨(0.71, 0.54), (0.63, 0.52)⟩	⟨(0.68, 0.53), (0.78, 0.45)⟩
Q_2	⟨(0.78, 0.59), (0.58, 0.65)⟩	⟨(0.74, 0.59), (0.58, 0.69)⟩	⟨(0.73, 0.60), (0.57, 0.54)⟩	⟨(0.64, 0.64), (0.80, 0.54)⟩
Q_3	⟨(0.74, 0.53), (0.61, 0.60)⟩	⟨(0.59, 0.69), (0.60, 0.63)⟩	⟨(0.65, 0.49), (0.77, 0.45)⟩	⟨(0.57, 0.63), (0.67, 0.57)⟩
Q_4	⟨(0.71, 0.65), (0.67, 0.70)⟩	⟨(0.57, 0.63), (0.66, 0.60)⟩	⟨(0.55, 0.70), (0.58, 0.71)⟩	⟨(0.79, 0.45), (0.59, 0.52)⟩
Q_5	⟨(0.63, 0.45), (0.57, 0.62)⟩	⟨(0.55, 0.59), (0.62, 0.55)⟩	⟨(0.72, 0.57), (0.79, 0.57)⟩	⟨(0.68, 0.69), (0.74, 0.57)⟩
Q_6	⟨(0.66, 0.47), (0.65, 0.71)⟩	⟨(0.72, 0.52), (0.65, 0.65)⟩	⟨(0.65, 0.46), (0.69, 0.63)⟩	⟨(0.63, 0.50), (0.61, 0.62)⟩
Q_7	⟨(0.53, 0.56), (0.79, 0.49)⟩	⟨(0.65, 0.73), (0.69, 0.60)⟩	⟨(0.58, 0.51), (0.66, 0.53)⟩	⟨(0.79, 0.51), (0.80, 0.52)⟩
Q_8	⟨(0.77, 0.57), (0.63, 0.60)⟩	⟨(0.61, 0.61), (0.70, 0.46)⟩	⟨(0.64, 0.73), (0.54, 0.65)⟩	⟨(0.63, 0.71), (0.80, 0.48)⟩
	T_5	T_6	T_7	T_8
Q_1	⟨(0.55, 0.56), (0.54, 0.71)⟩	⟨(0.69, 0.68), (0.71, 0.47)⟩	⟨(0.60, 0.75), (0.76, 0.48)⟩	⟨(0.64, 0.71), (0.65, 0.65)⟩
Q_2	⟨(0.60, 0.56), (0.72, 0.52)⟩	⟨(0.60, 0.75), (0.69, 0.49)⟩	⟨(0.61, 0.53), (0.71, 0.56)⟩	⟨(0.72, 0.52), (0.53, 0.56)⟩
Q_3	⟨(0.76, 0.60), (0.81, 0.54)⟩	⟨(0.73, 0.61), (0.73, 0.65)⟩	⟨(0.67, 0.56), (0.60, 0.72)⟩	⟨(0.60, 0.59), (0.61, 0.48)⟩
Q_4	⟨(0.62, 0.53), (0.63, 0.53)⟩	⟨(0.63, 0.45), (0.71, 0.57)⟩	⟨(0.71, 0.51), (0.63, 0.45)⟩	⟨(0.61, 0.67), (0.69, 0.44)⟩
Q_5	⟨(0.58, 0.57), (0.55, 0.58)⟩	⟨(0.60, 0.73), (0.56, 0.60)⟩	⟨(0.57, 0.60), (0.58, 0.58)⟩	⟨(0.79, 0.50), (0.53, 0.73)⟩
Q_6	⟨(0.68, 0.58), (0.63, 0.47)⟩	⟨(0.68, 0.66), (0.81, 0.46)⟩	⟨(0.63, 0.45), (0.54, 0.56)⟩	⟨(0.68, 0.60), (0.56, 0.68)⟩
Q_7	⟨(0.67, 0.56), (0.70, 0.67)⟩	⟨(0.73, 0.57), (0.74, 0.55)⟩	⟨(0.60, 0.51), (0.54, 0.54)⟩	⟨(0.70, 0.53), (0.55, 0.53)⟩
Q_8	⟨(0.68, 0.65), (0.62, 0.60)⟩	⟨(0.69, 0.51), (0.72, 0.66)⟩	⟨(0.57, 0.56), (0.77, 0.50)⟩	⟨(0.70, 0.64), (0.69, 0.49)⟩

Step 2. After normalizing the data in Table 4, Table 5 summarizes the outcomes.

Table 5
Normalized PFZN decision matrix PFZNIWG Operator

	T_1	T_2	T_3	T_4
Q_1	⟨(0.11, 0.17), (0.14, 0.12)⟩	⟨(0.14, 0.10), (0.12, 0.11)⟩	⟨(0.14, 0.12), (0.12, 0.11)⟩	⟨(0.13, 0.11), (0.13, 0.11)⟩
Q_2	⟨(0.14, 0.13), (0.11, 0.13)⟩	⟨(0.14, 0.12), (0.11, 0.15)⟩	⟨(0.14, 0.13), (0.11, 0.12)⟩	⟨(0.12, 0.14), (0.14, 0.13)⟩
Q_3	⟨(0.14, 0.12), (0.12, 0.12)⟩	⟨(0.11, 0.14), (0.12, 0.13)⟩	⟨(0.12, 0.11), (0.15, 0.10)⟩	⟨(0.11, 0.14), (0.12, 0.13)⟩
Q_4	⟨(0.13, 0.14), (0.13, 0.14)⟩	⟨(0.11, 0.13), (0.13, 0.13)⟩	⟨(0.11, 0.15), (0.11, 0.15)⟩	⟨(0.15, 0.10), (0.10, 0.12)⟩
Q_5	⟨(0.12, 0.10), (0.11, 0.12)⟩	⟨(0.11, 0.12), (0.12, 0.12)⟩	⟨(0.14, 0.12), (0.15, 0.12)⟩	⟨(0.13, 0.15), (0.13, 0.13)⟩
Q_6	⟨(0.12, 0.10), (0.12, 0.14)⟩	⟨(0.14, 0.11), (0.13, 0.14)⟩	⟨(0.12, 0.10), (0.13, 0.14)⟩	⟨(0.12, 0.11), (0.11, 0.15)⟩
Q_7	⟨(0.10, 0.12), (0.15, 0.10)⟩	⟨(0.13, 0.15), (0.14, 0.13)⟩	⟨(0.11, 0.11), (0.13, 0.12)⟩	⟨(0.15, 0.11), (0.14, 0.12)⟩
Q_8	⟨(0.14, 0.12), (0.12, 0.12)⟩	⟨(0.12, 0.13), (0.14, 0.10)⟩	⟨(0.12, 0.16), (0.10, 0.14)⟩	⟨(0.12, 0.15), (0.14, 0.11)⟩
	T_5	T_6	T_7	T_8
Q_1	⟨(0.11, 0.12), (0.10, 0.15)⟩	⟨(0.13, 0.14), (0.13, 0.11)⟩	⟨(0.12, 0.17), (0.15, 0.11)⟩	⟨(0.12, 0.15), (0.14, 0.14)⟩
Q_2	⟨(0.12, 0.12), (0.14, 0.11)⟩	⟨(0.11, 0.15), (0.12, 0.11)⟩	⟨(0.12, 0.12), (0.14, 0.13)⟩	⟨(0.13, 0.11), (0.11, 0.12)⟩
Q_3	⟨(0.15, 0.13), (0.16, 0.12)⟩	⟨(0.14, 0.12), (0.13, 0.15)⟩	⟨(0.14, 0.13), (0.12, 0.16)⟩	⟨(0.11, 0.12), (0.13, 0.11)⟩
Q_4	⟨(0.12, 0.11), (0.12, 0.11)⟩	⟨(0.12, 0.09), (0.13, 0.13)⟩	⟨(0.14, 0.11), (0.12, 0.10)⟩	⟨(0.11, 0.14), (0.14, 0.10)⟩
Q_5	⟨(0.11, 0.12), (0.11, 0.13)⟩	⟨(0.11, 0.15), (0.10, 0.13)⟩	⟨(0.11, 0.13), (0.11, 0.13)⟩	⟨(0.15, 0.11), (0.11, 0.16)⟩
Q_6	⟨(0.13, 0.13), (0.12, 0.10)⟩	⟨(0.13, 0.13), (0.14, 0.10)⟩	⟨(0.13, 0.10), (0.11, 0.13)⟩	⟨(0.13, 0.13), (0.12, 0.15)⟩
Q_7	⟨(0.13, 0.12), (0.13, 0.15)⟩	⟨(0.14, 0.11), (0.13, 0.12)⟩	⟨(0.12, 0.11), (0.11, 0.12)⟩	⟨(0.13, 0.11), (0.11, 0.12)⟩
Q_8	⟨(0.13, 0.14), (0.12, 0.13)⟩	⟨(0.13, 0.10), (0.13, 0.15)⟩	⟨(0.11, 0.13), (0.15, 0.11)⟩	⟨(0.13, 0.13), (0.14, 0.11)⟩

Step 3. Now evaluate T_i =PFZNIWG ($T_{i1}, T_{i2}, \dots, T_{ip}$) and we have the following Table 6.

Table 6
Evaluation by geometric operator

	T_1	T_2	T_3	T_4
Q_1	⟨(0.07, 0.10), (0.09, 0.07)⟩	⟨(0.08, 0.05), (0.06, 0.06)⟩	⟨(0.09, 0.08), (0.08, 0.07)⟩	⟨(0.09, 0.08), (0.09, 0.08)⟩
Q_2	⟨(0.09, 0.08), (0.07, 0.08)⟩	⟨(0.08, 0.06), (0.06, 0.08)⟩	⟨(0.09, 0.08), (0.07, 0.08)⟩	⟨(0.08, 0.10), (0.10, 0.09)⟩
Q_3	⟨(0.09, 0.07), (0.07, 0.07)⟩	⟨(0.06, 0.08), (0.06, 0.07)⟩	⟨(0.08, 0.07), (0.09, 0.06)⟩	⟨(0.08, 0.10), (0.08, 0.09)⟩
Q_4	⟨(0.08, 0.09), (0.08, 0.09)⟩	⟨(0.06, 0.07), (0.07, 0.07)⟩	⟨(0.07, 0.09), (0.07, 0.09)⟩	⟨(0.10, 0.07), (0.07, 0.08)⟩
Q_5	⟨(0.07, 0.06), (0.07, 0.07)⟩	⟨(0.06, 0.06), (0.06, 0.06)⟩	⟨(0.09, 0.08), (0.09, 0.08)⟩	⟨(0.09, 0.10), (0.09, 0.09)⟩
Q_6	⟨(0.07, 0.06), (0.07, 0.09)⟩	⟨(0.08, 0.06), (0.07, 0.08)⟩	⟨(0.08, 0.06), (0.08, 0.09)⟩	⟨(0.08, 0.08), (0.08, 0.10)⟩
Q_7	⟨(0.06, 0.07), (0.09, 0.06)⟩	⟨(0.07, 0.08), (0.08, 0.07)⟩	⟨(0.07, 0.07), (0.08, 0.08)⟩	⟨(0.10, 0.08), (0.10, 0.08)⟩
Q_8	⟨(0.09, 0.07), (0.07, 0.07)⟩	⟨(0.06, 0.07), (0.08, 0.05)⟩	⟨(0.08, 0.10), (0.06, 0.09)⟩	⟨(0.08, 0.10), (0.10, 0.08)⟩
	T_5	T_6	T_7	T_8
Q_1	⟨(0.06, 0.07), (0.06, 0.09)⟩	⟨(0.08, 0.09), (0.08, 0.07)⟩	⟨(0.07, 0.10), (0.08, 0.06)⟩	⟨(0.08, 0.10), (0.10, 0.10)⟩
Q_2	⟨(0.07, 0.07), (0.08, 0.06)⟩	⟨(0.07, 0.10), (0.08, 0.07)⟩	⟨(0.07, 0.07), (0.08, 0.07)⟩	⟨(0.09, 0.07), (0.07, 0.08)⟩
Q_3	⟨(0.09, 0.08), (0.09, 0.07)⟩	⟨(0.09, 0.08), (0.08, 0.10)⟩	⟨(0.08, 0.07), (0.07, 0.09)⟩	⟨(0.07, 0.08), (0.09, 0.07)⟩
Q_4	⟨(0.07, 0.06), (0.07, 0.06)⟩	⟨(0.08, 0.06), (0.08, 0.08)⟩	⟨(0.08, 0.06), (0.07, 0.06)⟩	⟨(0.07, 0.10), (0.10, 0.07)⟩
Q_5	⟨(0.06, 0.07), (0.06, 0.08)⟩	⟨(0.07, 0.10), (0.06, 0.08)⟩	⟨(0.06, 0.07), (0.06, 0.07)⟩	⟨(0.10, 0.07), (0.07, 0.11)⟩
Q_6	⟨(0.08, 0.08), (0.07, 0.06)⟩	⟨(0.08, 0.08), (0.09, 0.06)⟩	⟨(0.07, 0.06), (0.06, 0.07)⟩	⟨(0.09, 0.09), (0.08, 0.10)⟩
Q_7	⟨(0.08, 0.07), (0.08, 0.09)⟩	⟨(0.09, 0.07), (0.08, 0.08)⟩	⟨(0.07, 0.06), (0.06, 0.07)⟩	⟨(0.09, 0.07), (0.07, 0.08)⟩
Q_8	⟨(0.08, 0.08), (0.07, 0.08)⟩	⟨(0.08, 0.06), (0.08, 0.10)⟩	⟨(0.06, 0.07), (0.08, 0.06)⟩	⟨(0.09, 0.09), (0.10, 0.07)⟩

Step 4. After applying the proposed aggregation operators, the aggregated score matrix is obtained as follows:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
Q_1	0.5004	0.5002	0.5008	0.5000	0.4994	0.5008	0.5011	0.4990
Q_2	0.5008	0.5000	0.5008	0.4995	0.5001	0.5007	0.4997	0.5004
Q_3	0.5007	0.5003	0.5001	0.5004	0.5005	0.4996	0.4997	0.4997
Q_4	0.5000	0.4997	0.5000	0.5007	0.5000	0.4992	0.5003	0.5000
Q_5	0.4997	0.5000	0.5000	0.5005	0.4997	0.5011	0.5000	0.4997
Q_6	0.4990	0.4996	0.4988	0.4992	0.5011	0.5005	0.5000	0.5001
Q_7	0.4994	0.5000	0.4993	0.5000	0.4992	0.5000	0.5000	0.5004
Q_8	0.5007	0.5001	0.5013	0.5000	0.5004	0.4984	0.4997	0.5006

Determine the score functions for each and every Q_i, we have

$S(\mathrm{Q}_1)$ =0.500213, $S(\mathrm{Q}_2) = 0.500250, S(\mathrm{Q}_3) = 0.500125, S(\mathrm{Q}_4) = 0.499988,$
$S(\mathrm{Q}_5) = 0.500088, S(\mathrm{Q}_6) = 0.499788, S(\mathrm{Q}_7) = 0.499788, S(\mathrm{Q}_8) = 0.500150.$

Step 5. Sort all of the values Q_i $(i = 1,2,3, \dots p)$ based on the score function's values,

$$\mathrm{Q}_2 \succ \mathrm{Q}_1 \succ \mathrm{Q}_8 \succ \mathrm{Q}_3 \succ \mathrm{Q}_5 \succ \mathrm{Q}_4 \succ \mathrm{Q}_6 \succ \mathrm{Q}_7.$$

Here, Q_5 is the best option. To validate the effectiveness of the proposed PFZN MCDM model under high uncertainty, sensitivity tests of the input criteria and experts' opinions were conducted. The results show the model's robustness and reliability in identifying the best-ranked alternatives, even when the input data are highly uncertain. Figure 5 shows the ranking of alternatives.

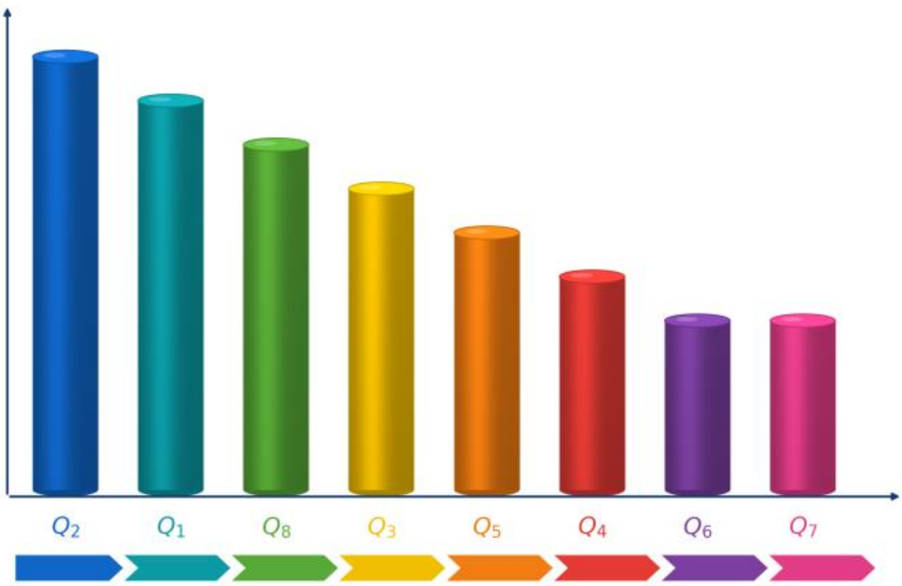

Fig. 5. Ranking of alternatives using geometric operators

5. Conclusion

To address uncertainty, vagueness, and reliability concerns in MCDM situations, this study develops an integrated decision-making framework that combines Z-number theory with PFZNs using Hamacher interactive aggregation operators. By accounting for both assessment values and information reliability, the suggested PFZN-based PFZNIWG operator provides an efficient method for combining expert judgments. By incorporating expert confidence and interaction effects among decision factors, this method improves the performance of conventional fuzzy decision-making techniques, making it more appropriate for complex, uncertain decision contexts.

The suggested framework is used to evaluate various development options based on a range of factors, including flood risk exposure, land suitability, infrastructure costs, population density, environmental sustainability, disaster resilience, accessibility, and socioeconomic development potential, in a real-world floodplain urban development assessment in Bangladesh. The findings show that, in uncertain, reliability-dependent situations, the proposed PFZN-based Hamacher interactive aggregation strategy efficiently prioritizes urban development options. The case study validates the applicability, stability, and resilience of the proposed model in supporting sustainable urban planning decisions that must simultaneously account for competing criteria, expert opinions, and environmental hazards.

By enhancing the representation of reluctant and reliability-based expert knowledge, offering a flexible aggregation strategy, and facilitating more reliable prioritization of urban development, the proposed framework advances decision-making under uncertainty. Nevertheless, the study has some drawbacks, such as a reliance on expert opinions that may vary with expertise and experience, and validation based on a single case study in Bangladesh. By adding real-time data sources such as GIS, remote sensing, and IoT-based flood monitoring systems, and by developing dynamic expert reliability models and hybrid aggregation operators, future research can expand on the suggested methodology. Overall, the recommended PFZN-based interactive aggregation framework is a powerful, useful tool for planning urban development in floodplains with a risk-aware, sustainable approach.

Author Contributions

Conceptualization, R.S.K. and F.M.; methodology, S.M.Q.; software, F.M., and N.K.; validation, R.S.K., N.K., and Z.Z.; formal analysis, S.M.Q.; investigation, R.S.K.; resources, F.M.; data curation, S.M.Q.; writing—original draft preparation, R.S.K.; writing—review and editing, S.M.Q.; visualization, F.M.; supervision, N.K.; project administration, F.M.; funding acquisition, N.K. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflicts of interest.

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