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A Novel T-Spherical Fuzzy Multi-Criteria Group Decision-Making for Air Quality Index Assessment

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ABSTRACT

In this study, we expose the fact that adequately assessing the air quality index in an atmosphere based on limited observations is essential for meeting environmental management goals. Fuzzy information has proven to be well-suited for handling uncertainty and subjectivity in ecological studies. Researchers use fuzzy set (FS) to address uncertainty in air quality indices. This article presents a novel decision-making approach that evaluates five alternatives against five attributes. For this purpose, decision-makers (Ɗ) used the T-Spherical FS (TSFS) framework in the assessment process. Additionally, we suggest prioritizing attribute weights based on their importance. Further, we consider T-spherical fuzzy weighted averaging (TSFWA) and T-spherical fuzzy weighted geometric (TSFWG) aggregation operators (AOs) within the framework of T-spherical fuzzy (TSF) information, and subsequently apply them to compute the weighted aggregated sum product assessment (WASPAS) technique. The proposed model handles uncertainties in hypothetical air quality index data, and we demonstrate its reliability through a case study and a sensitivity analysis that highlight its effectiveness and enable comparison with earlier work.

1. Introduction

A fuzzy set (FS) was first introduced by Zadeh [1], in which the membership degree (MD) is restricted to values less than or equal to 1. But the framework is not suitable for managing complex fuzzy information. To overcome this, Atanassov [2] introduced the intuitionistic fuzzy set (IFS), which includes a non-membership degree (NMD). Subsequent developments in this area include Yager's [3] Pythagorean FS, which is more flexible in handling uncertain information. Yager [4] extended the range of FS theory to develop the generalized orthopair FS (q-ROFS). In some settings, however, such as elections, where it is necessary to take four types of information into account: MD, abstinence degree (AD), NMD, and refusal degree (RD), the framework of q-ROFS is not sufficient to cover all of these. To further strengthen the power of FS theory, Cuong [5] introduced picture FS (PFS), an extension of IFS, and incorporated an AD into it. Mahmood et al. [6] proposed a new framework for

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handling uncertainty, called Spherical FS (SFS). But in SFS, the domain is limited, which reduces its applicability. SFS has several limitations, and to overcome this, Mahmood et al. [6] introduced TSFS. This framework provides a more robust and flexible approach to managing complex fuzzy information. The above frameworks and generalized FS extension are briefly discussed in Figure 1.

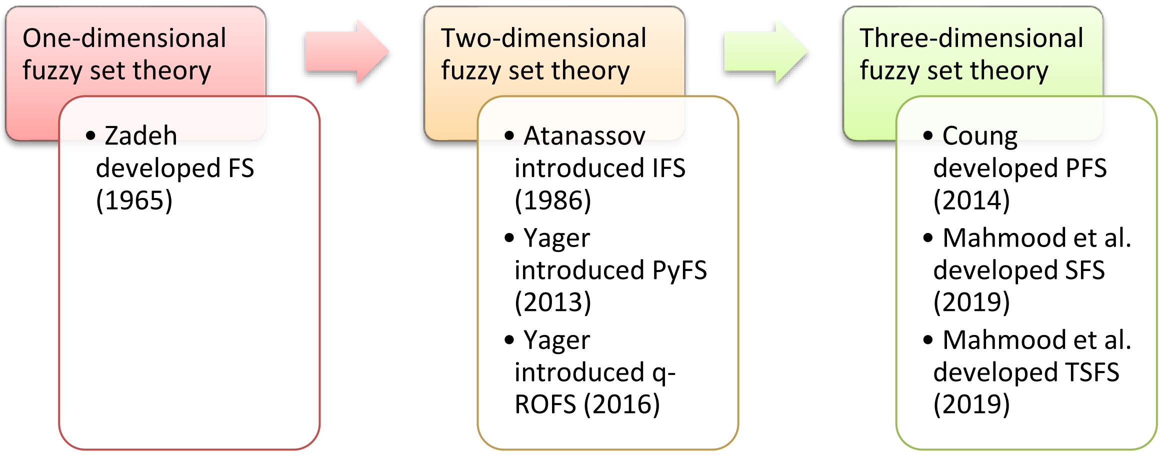

Fig. 1. Generalization of FS extension

Multi-attribute decision-making (MADM) enables Ɗ to identify trade-offs, assess risks, and optimize outcomes, thereby enhancing the quality and reliability of their decisions. As a result, MADM has been widely applied in various fields, including business, engineering, environmental management, and healthcare, to support strategic planning, resource allocation, and policy development. Hussain et al. [7] initiated Aczel-Alsina AOs in MADM; Ahmad et al. [8] investigated circular picture fuzzy Muirhead means for decision-making problems, and Seikh et al. [9] introduced AOs based on Frank t-norm and t-conorm in MADM. This approach is particularly useful in scenarios where multiple attributes are considered by a group of Ɗ, transforming the MADM approach into a multi-attribute group decision-making (MAGDM) approach. Akram et al. [10] developed group decision-making based on a complex spherical fuzzy VIKOR approach. Ullah et al. [11] developed some circular Pythagorean fuzzy Muirhead means for the MAGDM problem. Hussain et al. [12] developed a MAGDM method based on the Pythagorean fuzzy rough set and the novel Schweizer-Sklar t-norm and t-conorm. Ashraf et al. [13] designed different approaches to the MAGDM problem for PFS. Haleemzai et al. [14] introduced a circular q-rung orthopair fuzzy WASPAS method for assessing the role of the digital economy in growth.

1.1. Motivation

To improve air quality, a comprehensive strategy is needed to reduce levels of harmful pollutants, including nitrogen dioxide, ozone, sulfur dioxide, particulate matter, and carbon monoxide. These wastes may prove to be disastrous to human health and the environment. The strategies to be used to reduce their effects are to maximize the effective strategies, which include:

- Technological innovation: Implementing cutting-edge technologies that minimize emissions and optimize energy efficiency.
- Policy enforcement: Enacting and enforcing stringent regulations to limit pollution from various sources.
- Societal shifts: Promoting behavioral changes that reduce emissions, such as using public transport, carpooling, or driving electric vehicles.
- Public awareness: Educating the public about the importance of air quality and the simple actions they can take to contribute to a cleaner environment.

All these strategies can be used together to drastically enhance air quality, lower the air quality index, and provide a better living environment for urban and rural communities. It's all about the little things, and together they can make a difference for a cleaner, more sustainable environment. Many methods are available to improve air quality and reduce the air quality index. It includes:

- Transportation: Technology and incentivizing electric or hybrid vehicles, improving public transportation, and promoting active transportation (walking or cycling).
- Agriculture: Adopting sustainable agricultural methods to reduce emissions and the use of polluting pesticides and fertilizers.
- Industry: Implementing cleaner production techniques, investing in pollution-reducing technologies, and implementing strict emission controls.
- Urban planning: Planning cities and urban spaces with a focus on improving green infrastructure, pedestrian facilities, and public transport systems.

Each sector plays an important role in providing a cleaner, healthier environment. In addition to these strategies, we can reduce air pollution and enhance the overall quality of life for current and future generations. From the research perspective, several notable contributions can be highlighted. Sowlat et al. [15] elaborated a fuzzy-based air quality index, while Mandal et al. [16] developed a fuzzy air quality index using a soft computing approach. Wang et al. [17] introduced a fuzzy expression framework for the air quality index, and Chenyang et al. [18] investigated the use of a hesitant fuzzy environment and its application to the air quality index.

1.2. Literature Review

The WASPAS technique offers a significant benefit for addressing complex decision-making questions that entail both benefits and costs, which are often interrelated and impactful. For example, in the air quality index, decision-making can yield greater benefits and qualities, and attention to attribute relations is needed to improve accuracy and efficiency. Nevertheless, the current AOs utilizing the WASPAS methodology are constrained by TSFS. We overcome this limitation by creating the WASPAS method within the TSF framework, enabling it to address complex problems using TSF information. Implementing the WASPAS method within the TSF framework will enable more precise and effective decision-making and open the way to novel applications across many fields.

This research manuscript extends the widely used MAGDM method, the WASPAS technique, to the TSF version. The weighted sum model (WSM) and the weighted product model (WPM) are combined in the WASPAS by Zavadskas et al. [19] to achieve a detailed assessment of the alternatives. The WASPAS has been favored for its ability to address complex decision-making issues, with both qualitative and quantitative elements. The manuscript proposes a novel application of the WASPAS method within the TSFS framework to develop an air quality index that effectively addresses the complex decision-making problem. A summary of the WASPAS model across various frameworks is presented in Table 1, highlighting its potential uses and advantages across many areas.

Table 1
Previous work on the WASPAS model

Year	References	Framework	Supporting approach	Application
2018	Stanujkic et al. [20]	IFS	MADM	Website evaluation
2019	Kutlu et al. [21]	SFS	MAGDM	Illustrative example
2020	Otay et al.[22]	Single-valued SFS	MADM	Illustrative example
2021	Simic et al. [23]	PFS	MADM	Study of Belgrade
2022	Wei et al. [24]	PyFS	MAGDM	Teaching quality
2023	Rao et al. [25]	Fermatean Fuzzy	MADM	Healthcare
2024	Wang et al. [26]	q-ROFS	MAGDM	Community group
2026	Initiated work	TSFS	MAGDM	Air quality index

1.3. Research Gap and Research Questions

TSFS is one of the most important extensions of FS theory, a general model that encompasses all other fuzzy models. Many of the operators and methodological improvements are formulated in TSFS, as it can handle the complexity of decision uncertainty. However, the literature search and analysis showed potential for significant improvements and extensions to the existing methods. In particular, the following are the primary problems that remain in research work:

1. How can Ɗ effectively represent and evaluate alternatives to determine the most appropriate opinion for a given decision problem?
2. How can normalized decision matrices be constructed when dealing with both beneficial and cost attributes?
3. How can the weight vector be prioritized to reflect the relative importance of various attributes accurately?
4. How can the WASPAS technique be reformulated to enable efficient aggregation under the TSFS framework?
5. How can data about alternatives and their associated attributes be structured into singleton sets using suitable AOs?
6. How can the optimal option be selected from a finite set of alternatives in environments characterized by uncertainty?

These challenges are directly considered in the proposed work introduced in this manuscript, which offers a comprehensive and flexible approach in the context of the TSFS. Moreover, the TSFS environment will be more flexible and reliable with the addition of the WASPAS model to the MAGDM approach for handling uncertainty. Moreover, the current methods are insufficient to provide the depth required to assess alternatives in terms of attribute importance and weight prioritization in a TSFS context, as shown in Table 1. In this regard, the offer of such a model fills a gap, providing an additional, structured mechanism to improve decision-making while allowing the use of a wider range of uncertain data. Finally, the paper not only applies the WASPAS model to a TSFS model but also makes a significant contribution to overcoming the drawbacks of the presented fuzzy MAGDM model.

1.4. Contribution

This research is a notable theoretical contribution to solving complex MAGDM problems under uncertainty by extending the WASPAS method within the TSFS environment. Some models based on intuitionistic fuzzy, Pythagorean fuzzy, spherical fuzzy, picture fuzzy, Fermatean fuzzy, and q-rung orthopair fuzzy environments have been developed for TSF. Still, no comprehensive model exists in the WASPAS fuzzy environment. The proposed model combines the flexibility of TSFS with the hybrid decision-making of WASPAS, providing a more expressive and general way to represent membership, abstention, and non-membership information, thereby facilitating decision analysis under high uncertainty. One of the significant results of this research is the design of a systematic MAGDM algorithm that incorporates decision-makers' experience levels and attribute weights prioritized in the evaluation process. The proposed methodology includes procedures for treating both benefit and cost attributes by normalizing them, aggregating experts' opinions using TSF-weighted averaging (TSFWA) and TSF-weighted geometric (TSFWG) aggregation operators, and, finally, combining weighted sum and weighted product models using the WASPAS framework. This holistic process not only improves decision-making accuracy but also considers the significance of attributes and the reliability of experts.

The study illustrates the usefulness of the suggested framework by comparing the proposed alternatives to the air quality index with five key environmental attributes. This research does not represent a theoretical development process but is a practical application of the environmental decision-making process, with many experts comparing various air quality index models. The results indicate that the most suitable approach is the air quality health index, and the proposed method can assist in environmental planning and sustainable policy-making, even in a scenario with complex, uncertain data. Moreover, the article has contributed to the literature by conducting a sensitivity analysis and a comparative evaluation to ensure robustness. The sensitivity analysis results indicate that the rank of the alternatives does not change significantly across different parameter variations, illustrating the consistency and reliability of the model. Besides, the proposed TSF-WASPAS is validated by comparing it with other available aggregation operators, such as T-spherical fuzzy operators like the Einstein operator, Interaction operator, and Hamacher operator, to show that the TSF-WASPAS framework is more flexible, with a hybrid aggregation mechanism, and yields competitive and consistent ranking results. The proposed methodology not only can make contributions to the theoretical aspects of fuzzy decision-making but also is a practical and powerful decision-supporting tool for solving a MAGDM problem with uncertainty and imprecise information.

1.5. Structure of the Article

The article is structured as follows. Section 1 provides the introduction, setting the context and motivation for the study. Section 2 recalls TSFS, basic operations, and AOs that are referenced throughout the article. In Section 3, the WASPAS model algorithm and methodology are presented within the TSFS framework. Section 4 illustrates the practical application of the initiated work through a case study. Section 5 examines the sensitivity, while Section 6 compares the new operators with existing approaches. Section 7 presents the conclusion of the manuscript.

2. Background

This section aims to discuss the concept of TSFS and introduce the basic operational laws required for this article. Further, the TSFWA operator (TSFWAO), the TSFWG operator (TSFWGO), the score value, and the accuracy values of TSFS are recalled.

Definition 1 [6]**:** Let X be the fixed universal set. For any TSFS β on the fixed universal set X is define as $\beta = \{< \mathfrak{x}, \mathfrak{m}(\mathfrak{x}), \mathfrak{a}(\mathfrak{x}), \mathfrak{n}(\mathfrak{x}) > | \mathfrak{x} \in X\}$, where the functions are $\mathfrak{m}: X \to [0,1]$, $\mathfrak{a}: X \to [0,1]$, and $\mathfrak{n}: X \to [0,1]$ MD, AD, and NMD provided that $0 \leq \mathfrak{m}^t(\mathfrak{x}) + \mathfrak{a}^t(\mathfrak{x}) + \mathfrak{n}^t(\mathfrak{x}) \leq 1$, for some $t \in \mathbb{Z}^+$. The RD is $\mathfrak{r}(\mathfrak{x}) = \sqrt{1 - \left(\mathfrak{m}^t(\mathfrak{x}) + \mathfrak{a}^t(\mathfrak{x}) + \mathfrak{n}^t(\mathfrak{x})\right)}$ of $\mathfrak{x} \in X$ in β. For convenience, $\beta_J = (\mathfrak{m}_J, \mathfrak{a}_J, \mathfrak{n}_J)$, $(J = 1,2, \dots, \mathfrak{y})$ is known as a TSF value (TSFV).

Definition 2 [6]**:** Let $\beta_1 = (\mathfrak{m}_1, \mathfrak{a}_1, \mathfrak{n}_1)$ and $\beta_2 = (\mathfrak{m}_2, \mathfrak{a}_2, \mathfrak{n}_2)$ be the two TSFVs on the fixed universal set X and each value is characterized between 0 and 1 independently. Then the operation between them is defined as:

$$\beta_1 \oplus \beta_2 = \left\{\sqrt[t]{\mathfrak{m}^t{}_1 + \mathfrak{m}^t{}_2 - \mathfrak{m}^t{}_1 \mathfrak{m}^t{}_2}), \mathfrak{a}_1 \mathfrak{a}_2, \mathfrak{n}_1 \mathfrak{n}_2\right\} \tag{1}$$

$$\beta_1 \otimes \beta_2 = \left\{\mathfrak{m}_1 \mathfrak{m}_2, \mathfrak{a}_1 \mathfrak{a}_2, \sqrt[t]{\mathfrak{n}^t{}_1 + \mathfrak{n}^t{}_2 - \mathfrak{n}^t{}_1 \mathfrak{n}^t{}_2})\right\} \tag{2}$$

$$\lambda \beta_1 = \left\{\sqrt[t]{1 - (1 - \mathfrak{m}^t{}_1)^{\lambda}}, \mathfrak{a}_1{}^{\lambda}, \mathfrak{n}_1{}^{\lambda}\right\} \tag{3}$$

$$\beta_1{}^{\lambda} = \left\{\mathfrak{m}_1{}^{\lambda}, \mathfrak{a}_1{}^{\lambda}, \sqrt[t]{1 - (1 - \mathfrak{n}^t{}_1)^{\lambda}}\right\} \tag{4}$$

$$\beta_1^c = \{\mathfrak{n}_1, \mathfrak{a}_1, \mathfrak{m}_1\} \tag{5}$$

Definition 3 [27]**:** The TSFVs β_J have corresponding weights ω_J such that $(J = 1,2,3, \dots, \mathfrak{y})$ and the weights satisfied the constraint $\omega_J \geq 0$ and $\sum \omega_J = 1$. Then, the TSFWAO is of the form:

$$TSFWAO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \sum_{J=1}^{\mathfrak{y}} \omega_J \beta_J \tag{6}$$

The aggregated value of $\mathrm{A_J}$ by utilizing the TSFWAO is also a TSFV and is given below:

$$TSFWAO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \left(\sqrt[t]{1 - \prod_{J=1}^{\mathfrak{y}} (1 - \mathfrak{m}^t{}_J)^{\omega_J}}, \prod_{J=1}^{\mathfrak{y}} \mathfrak{a}_J{}^{\omega_J}, \prod_{J=1}^{\mathfrak{y}} \mathfrak{n}_J{}^{\omega_J}\right) \tag{7}$$

Definition 4 [27]**:** The TSFVs β_J have corresponding weights ω_J such that $(J = 1,2,3, \dots, \mathfrak{y})$ and the weights satisfied the constraint $\omega_J \geq 0$ and $\sum \omega_J = 1$. Then, the TSFWGO is of the form:

$$TSFWGO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \sum_{J=1}^{\mathfrak{y}} \beta_J{}^{\omega_J} \tag{8}$$

The aggregated value of $\mathrm{A_J}$ by utilizing the TSFWAO is also a TSFV and is given below:

$$TSFWGO(\beta_1, \beta_2, \beta_3, \dots, \beta_J) = \left(\prod_{J=1}^{\mathfrak{y}} \mathfrak{m}_J{}^{\omega_J}, \prod_{J=1}^{\mathfrak{y}} \mathfrak{a}_J{}^{\omega_J}, \sqrt[t]{1 - \prod_{J=1}^{\mathfrak{y}} (1 - \mathfrak{n}^t{}_J)^{\omega_J}}\right) \tag{9}$$

Definition 5 [27]**:** For any TSFV $\beta = (\mathfrak{m}, \mathfrak{a}, \mathfrak{n})$, the score value of β denoted by (Š) and accuracy value of β denoted by (Ă) are given by

$$\check{S}(\beta) = \mathfrak{m}^t - \mathfrak{n}^t \in [-1,1] \tag{10}$$

$$\breve{\mathrm{A}}(\beta) = \mathrm{m}^t + \mathrm{a}^t + \mathrm{n}^t \in [0,1] \tag{11}$$

3. WASPAS Model Algorithm in the Environment of T-Spherical Fuzzy Set

This section explores the WASPAS method. This hybrid approach seamlessly combines the strengths of the weighted-sum and weighted-product models, tailored for TSF data environments. The WASPAS technique has a history of success that has been well embraced across many areas of application, such as engineering, computer science, and business management, and continues to prove itself a versatile and efficient tool for solving complex decision-making problems. The suggested model has several benefits; specifically, the WASPAS method can efficiently address issues that entail both utility and positive qualities. As opposed to other FS methods (e.g., SFS), the WASPAS method does not demand that the Ɗ assign MD, AD, and NMD, with their squared sum being at most equal to 1. Rather, the size of the power of its degrees of $t \geq 2$ and above is not limited, which means that it is more adaptable to complex decision-making problems.

3.1. Methodology for MAGDM Approach

In this subsection, we introduce a MAGDM framework to address complex, uncertain problems in a TSF information environment. To validate the efficacy and superiority of our proposed models, we employ the MAGDM methodology in conjunction with the TSFWAO and TSFWGO. This approach enables us to effectively handle TSF information and demonstrate the robustness of our proposed models in tackling decision-making problems:

Step 1: The kth Ɗ investigates m̦ alternatives against ŋ attributes whose decision values are in the form of TSF versions and stored in the DM, as showcased in Eq. (12).

$$M^{k}{}_{\text{m̦}\times\text{ŋ}} = \begin{array}{c} \\ \mathring{\mathrm{A}}_1 \\ \mathring{\mathrm{A}}_2 \\ \vdots \\ \mathring{\mathrm{A}}_{\text{m̦}} \end{array} \begin{bmatrix} \hat{\mathrm{C}}_1 & \hat{\mathrm{C}}_2 & \dots & \hat{\mathrm{C}}_{\text{ŋ}} \\ \mathfrak{x}^{k}{}_{11} & \mathfrak{x}^{k}{}_{12} & \dots & \mathfrak{x}^{k}{}_{1\text{ŋ}} \\ \mathfrak{x}^{k}{}_{21} & \mathfrak{x}^{k}{}_{22} & \dots & \mathfrak{x}^{k}{}_{2\text{ŋ}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathfrak{x}^{k}{}_{\text{m̦}1} & \mathfrak{x}^{k}{}_{\text{m̦}2} & \dots & \mathfrak{x}^{k}{}_{\text{m̦ŋ}} \end{bmatrix} \tag{12}$$

Step 2: In case the DM has a cost attribute. Then, we should normalize the DM using Eq. (13) and Eq. (14), and the normalized DM is given by Eq. (15).

$$\tilde{\mathfrak{x}}^{k}{}_{\ddot{\imath}\jmath} = \begin{cases} (\mathrm{m}_{\ddot{\imath}\jmath}, \mathrm{a}_{\ddot{\imath}\jmath}, \mathrm{n}_{\ddot{\imath}\,\boldsymbol{\jmath}\,\ddot{\imath}\jmath}); & \text{for benefit attribute} \\ (\mathrm{n}_{\ddot{\imath}\,\boldsymbol{\jmath}\,\ddot{\imath}\jmath}, \mathrm{a}_{\ddot{\imath}\jmath}, \mathrm{m}_{\ddot{\imath}\jmath}); & \text{for cost attribute} \end{cases} \tag{13}$$

(14)

$$\widetilde{M}^{k}{}_{\text{m̦}\times\text{ŋ}} = \begin{array}{c} \\ \mathring{\mathrm{A}}_1 \\ \mathring{\mathrm{A}}_2 \\ \vdots \\ \mathring{\mathrm{A}}_{\text{m̦}} \end{array} \begin{bmatrix} \hat{\mathrm{C}}_1 & \hat{\mathrm{C}}_2 & \dots & \hat{\mathrm{C}}_{\text{ŋ}} \\ \tilde{\mathfrak{x}}^{k}{}_{11} & \tilde{\mathfrak{x}}^{k}{}_{12} & \dots & \tilde{\mathfrak{x}}^{k}{}_{1\text{ŋ}} \\ \tilde{\mathfrak{x}}^{k}{}_{21} & \tilde{\mathfrak{x}}^{k}{}_{22} & \dots & \tilde{\mathfrak{x}}^{k}{}_{1\text{ŋ}} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathfrak{x}}^{k}{}_{\text{m̦}1} & \tilde{\mathfrak{x}}^{k}{}_{\text{m̦}2} & \dots & \tilde{\mathfrak{x}}^{k}{}_{\text{m̦ŋ}} \end{bmatrix} \tag{15}$$

Step 3.1: Prioritization of attribute importance weights, recognizing that attribute weights may differ. To accurately identify attribute weights, it is essential to aggregate the opinions of Ɗ regarding the importance of each attribute. This is achieved by utilizing TSF information, where Ɗ provides their assessments of attribute importance, which are then aggregated using a suitable approach, as outlined in the following DM presented in Eq. (16).

$$\omega = \begin{array}{c} \\ \mathrm{D}_1 \\ \mathrm{D}_2 \\ \vdots \\ \mathrm{D}_k \end{array} \begin{bmatrix} \hat{C}_1 & \hat{C}_2 & \dots & \hat{C}_{\eta} \\ x_{11} & x_{12} & \dots & x_{1\eta} \\ x_{21} & x_{22} & \dots & x_{2\eta} \\ \vdots & \vdots & \ddots & \vdots \\ x_{k1} & x_{k2} & \dots & x_{k\eta} \end{bmatrix} \tag{16}$$

Step 3.2: Next, we aggregate the DM using Eq. (7), considering the weights of kth D with different experience levels. The aggregated attribute weights are then defuzzified (denoted by $\mathfrak{D}$) using Eq. (17) and normalized using Eq. (18) to derive the final weights. It is necessary that each prioritized weight value is greater than zero and that their sum is equal to one.

$$\mathfrak{D}_J = \mathfrak{m}_J^t - \mathfrak{n}_J^t \tag{17}$$

$$\omega_J = \frac{\mathfrak{D}_J}{\sum_{J=1}^{\eta} \mathfrak{D}_J} \tag{18}$$

Step 4.1: Aggregate the DM presented in Eq. (15) by using Eq. (7) with the help of experience levels of kth D, and the aggregated DM is given in Eq. (19).

$$\widetilde{M}_{m\times\eta} = \begin{array}{c} \\ \mathring{A}_1 \\ \mathring{A}_2 \\ \vdots \\ \mathring{A}_{m} \end{array} \begin{bmatrix} \hat{C}_1 & \hat{C}_2 & \dots & \hat{C}_{\eta} \\ \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1\eta} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{1\eta} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & x_{m2} & \dots & \tilde{x}_{m\eta} \end{bmatrix} \tag{19}$$

Step 4.2: According to the WASPAS method, the weighted sum model in the TSF framework is TSFWA, denoted by $Q_{\ddot{I}}^{S}$ and presented in Eq. (20). The first part, Eq. (20), is a multiplication operation, and the second part is an addition operation, which are performed. Both parts are calculated using Eq. (7), as shown in Eq. (21).

$$Q_{\ddot{I}}^{S} = TSFWA = \sum_{J=1}^{\eta} \widetilde{x_{\ddot{I}J}}\, \omega_J \tag{20}$$

$$Q_{\ddot{I}}^{S} = TSFWA = \left(\sqrt[t]{1 - \prod_{J=1}^{\eta} (1 - \mathfrak{m}^t{}_J)^{\omega_J}}, \prod_{J=1}^{\eta} \mathfrak{a}_J{}^{\omega_J}, \prod_{J=1}^{\eta} \mathfrak{n}_J{}^{\omega_J} \right) \tag{21}$$

Step 4.3: According to the WASPAS technique, the weighted product model in the TSF framework is TSFWG, denoted by $Q_{\ddot{I}}^{P}$ and presented in Eq. (22). The first part of Eq. (22) is an exponential operation, and the second part is a multiplication operation, which are performed. Both parts are calculated using Eq. (9), as shown in Eq. (23).

$$Q_{\ddot{I}}^{P} = TSFWG = \prod_{J=1}^{\eta} \widetilde{x_{\ddot{I}J}}^{\omega_J} \tag{22}$$

$$Q_{\ddot{I}}^{P} = TSFWG = \left(\prod_{J=1}^{\eta} \mathfrak{m}_J{}^{\omega_J}, \prod_{J=1}^{\eta} \mathfrak{a}_J{}^{\omega_J}, \sqrt[t]{1 - \prod_{J=1}^{\eta} (1 - \mathfrak{n}^t{}_J)^{\omega_J}} \right) \tag{23}$$

Step 4.4: Utilize Eq. (21) and Eq. (23) to aggregate the DM presented in Eq. (19) concerning prioritized weights, and then we will get $Q_{\ddot{I}}^{S}$ and $Q_{\ddot{I}}^{P}$ respectively. Use the convex formula to get the WASPAS model in the TSF framework, denoted by $Q_{\ddot{I}}$ and stored in Eq. (24). For the evaluation of Eq.

(24), both terms of Eq. (24) have a scalar multiplication operation, and are performed based on Eq. (3). Then, we utilize the addition operation defined in Eq. (1), and we get the DM showcased in Eq. (24).

$$Q_{\ddot{I}} = WASPAS = \mathbb{C}Q_{\ddot{I}}^{S} + (1 - \mathbb{C})Q_{\ddot{I}}^{P} \tag{24}$$

Step 5: To determine the ranking of alternatives, we defuzzify the $Q_{\ddot{I}}$ values using Eq. (10) and calculate the score values. The score values are then arranged in descending (or increasing) order, depending on the context. If two or more alternatives have the same score, we compute the accuracy using Eq. (11) to break the tie. Finally, based on the score and accuracy values (if needed), we rank the alternatives and select the most suitable one.

For better comprehension and visualization, the flowchart of the initiated work is shown in Figure 2.

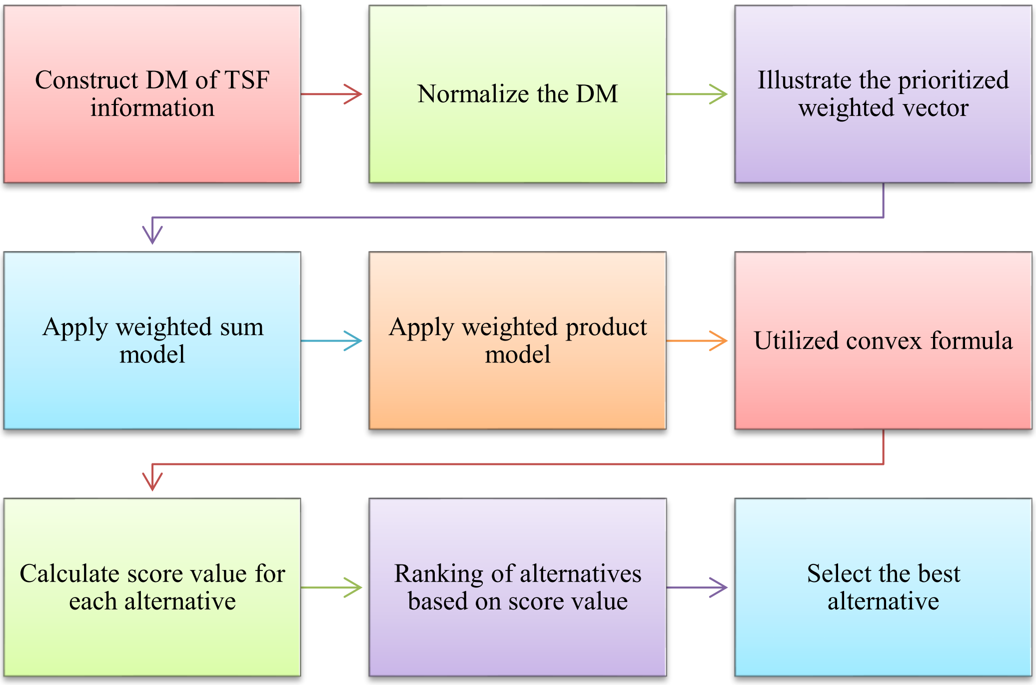

Fig.2. WASPAS Methodology for MAGDM Problems

4. Case Study

This section tackles the complexities of the air quality index by presenting the novel WASPAS technique within the TSF information framework, which is thoroughly examined in Section 3.

4.1. Alternatives

Consider five notable alternatives defined below:

1. Air Quality Health Index ($\mathring{A}_1$): A health-focused index that communicates the short-term health risks associated with air pollution. Unlike the traditional air quality index, which emphasizes pollution levels, AQHI estimates how current air quality will affect your health, especially for vulnerable groups such as children, older people, and people with respiratory conditions.

- Pollutants used: O_3, NO_2, PM2.5.
- Scale: 1 (low risk) to 10 plus (very high risk).

2. Common Air Quality Index ($\mathring{A}_2$): A European-standard air quality index that allows comparison of air pollution levels across EU countries. It's divided into hourly and daily values, making it useful for both short-term awareness and long-term trends.
   - Pollutants used: PM10, NO_2, O_3 (mandatory); PM2.5, SO_2, CO (optional).
   - Scale: 0 (very low) to 100 plus (very high).
3. World Air Quality Index ($\mathring{A}_3$): A global, real-time aggregation system that collects data from thousands of air quality monitoring stations worldwide. It is not an index in the scientific sense, but a platform that translates various national air quality indices into a unified format for global use.
   - Pollutants used: Vary depending on the country/station.
   - Access: Web, mobile app, and API.
4. Integrated Exposure–Response ($\mathring{A}_4$): A research-based model used in public health studies to estimate the risk of disease or death from long-term exposure to air pollution. It combines data from ambient, household, and smoking sources to model how increasing PM2.5 levels correlate with mortality and morbidity.
   - Pollutants used: PM2.5 (mainly).
   - Output: Health risk estimates, not public-scale indexes.
5. NowCast ($\mathring{A}_5$): An algorithm developed by the U.S. EPA that calculates real-time PM2.5 and PM10 values by giving more weight to recent pollution levels. It's essentially a dynamic air quality index that adjusts more quickly to spikes (like those from wildfires or traffic surges), rather than waiting for 24-hour averages.
   - Pollutants used: Primarily PM2.5 and PM10.
   - Purpose: Real-time air quality adjustment.
   -

4.2. Definition of Supposed Attributes

The five best attributes for evaluating the alternatives are given below:

1. Health Relevance ($\hat{C}_1$): Health relevance refers to how accurately an air quality index reflects the actual health risks posed by air pollution. This is especially important for sensitive groups such as children, the elderly, or people with asthma, heart disease, or respiratory conditions.
2. Responsiveness ($\hat{C}_2$): This refers to how quickly the index updates to reflect current air quality conditions, especially during rapidly changing events like wildfires, traffic spikes, or industrial emissions.
3. Geographic Coverage ($\hat{C}_3$): Geographic coverage reflects the extent to which an air quality index is usable or available across different regions, countries, or continents.
4. Data Transparency ($\hat{C}_4$): Data transparency refers to how openly an index or system shares its data sources, calculation methods, and assumptions. It also includes whether the raw data is publicly accessible.
5. Predictive Capability ($\hat{C}_5$): This refers to whether the system provides forecasts of future air quality conditions based on models, weather patterns, or historical data, typically for the next few hours or days.

4.3. Analyzing the Results

Let $\{\mathring{A}_{\mathbf{1}}, \mathring{A}_{\mathbf{2}}, \mathring{A}_{\mathbf{3}}, \mathring{A}_{\mathbf{4}}, \mathring{A}_{\mathbf{5}}\}$ and $\Psi = \{\hat{C}_{\mathbf{1}}, \hat{C}_{\mathbf{2}}, \hat{C}_{\mathbf{3}}, \hat{C}_{\mathbf{4}}, \hat{C}_{\mathbf{5}}\}$ be the alternative and the corresponding attribute, respectively. The Ɗ uses the mentioned attribute to evaluate the alternatives under consideration, with prioritized weights $(\omega_{\mathbf{1}}, \omega_{\mathbf{2}}, \dots, \omega_{\mathbf{5}})$ of all attributes, satisfying the constraint $\omega_{\boldsymbol{J}} \geq \mathbf{0}$ and $\sum \omega_{\boldsymbol{J}} = \mathbf{1}$ such that $(\boldsymbol{J} = \mathbf{1}, \mathbf{2}, \dots, \mathbf{5})$. Note that the data assumed in this study are hypothetical.

Step 1: The decision values for the Ɗ cover the air quality index, with the first, second, and fourth attributes benefits, and the third and fifth attributes costs. For this constraint, the data is in the form of TSFV with $t = 3$, which is stored in Table 2.

Table 2
DM of TSF information for the air quality index by the Ɗ

Ɗ	Å/Ĉ	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$Ɗ_1$	$\mathring{A}_1$	(0.90,0.10,0.20)	(0.80,0.30,0.40)	(0.55,0.65,0.75)	(0.80,0.30,0.40)	(0.50,0.70,0.80)
	$\mathring{A}_2$	(0.90,0.10,0.20)	(0.85,0.20,0.30)	(0.50,0.70,0.80)	(0.85,0.20,0.30)	(0.70,0.50,0.60)
	$\mathring{A}_3$	(0.80,0.30,0.40)	(0.75,0.40,0.50)	(0.70,0.50,0.60)	(0.90,0.10,0.20)	(0.80,0.30,0.40)
	$\mathring{A}_4$	(0.70,0.50,0.60)	(0.90,0.10,0.20)	(0.60,0.60,0.70)	(0.70,0.50,0.60)	(0.65,0.55,0.65)
	$\mathring{A}_5$	(0.85,0.20,0.30)	(0.75,0.40,0.50)	(0.55,0.65,0.75)	(0.75,0.40,0.50)	(0.75,0.40,0.50)
$Ɗ_2$	$\mathring{A}_1$	(0.85,0.20,0.30)	(0.80,0.30,0.40)	(0.60,0.60,0.70)	(0.90,0.10,0.20)	(0.50,0.70,0.80)
	$\mathring{A}_2$	(0.80,0.30,0.40)	(0.85,0.20,0.30)	(0.70,0.50,0.60)	(0.80,0.30,0.40)	(0.50,0.70,0.80)
	$\mathring{A}_3$	(0.90,0.10,0.20)	(0.70,0.50,0.60)	(0.65,0.55,0.65)	(0.90,0.10,0.20)	(0.60,0.60,0.70)
	$\mathring{A}_4$	(0.80,0.30,0.40)	(0.90,0.10,0.20)	(0.55,0.65,0.75)	(0.85,0.20,0.30)	(0.70,0.50,0.60)
	$\mathring{A}_5$	(0.85,0.20,0.30)	(0.80,0.30,0.40)	(0.50,0.70,0.80)	(0.85,0.20,0.30)	(0.80,0.30,0.40)
$Ɗ_3$	$\mathring{A}_1$	(0.75,0.40,0.50)	(0.80,0.30,0.40)	(0.70,0.50,0.60)	(0.75,0.40,0.50)	(0.55,0.65,0.75)
	$\mathring{A}_2$	(0.75,0.40,0.50)	(0.75,0.40,0.50)	(0.70,0.50,0.60)	(0.85,0.20,0.30)	(0.65,0.55,0.65)
	$\mathring{A}_3$	(0.80,0.30,0.40)	(0.75,0.40,0.50)	(0.50,0.70,0.80)	(0.90,0.10,0.20)	(0.70,0.50,0.60)
	$\mathring{A}_4$	(0.80,0.30,0.40)	(0.85,0.20,0.30)	(0.55,0.65,0.75)	(0.80,0.30,0.40)	(0.50,0.70,0.80)
	$\mathring{A}_5$	(0.90,0.10,0.20)	(0.85,0.20,0.30)	(0.60,0.60,0.70)	(0.70,0.50,0.60)	(0.65,0.55,0.65)

Step 2: The normalized DM is obtained using Eqs. (13) and (14) and is shown in Table 3.

Step 3.1 to Step 3.2: The prioritization of weights for attribute importance is determined using TSF information with $t = 3$, as outlined in Table 4. The weights for the three Ɗ, representing different experience levels, are assigned as follows: 0.38, 0.30, and 0.32, respectively. Aggregate Table 4 concerning experience levels of Ɗ based on Eq. (7). Then defuzzified and normalized, yielding the final attribute weights presented in Table 5.

Table 3
Normalized DM of the air quality index

Ɗ	Å/Ĉ	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$Ɗ_1$	$\mathring{A}_1$	(0.90,0.10,0.20)	(0.80,0.30,0.40)	(0.75,0.65,0.55)	(0.80,0.30,0.40)	(0.80,0.70,0.50)
	$\mathring{A}_2$	(0.90,0.10,0.20)	(0.85,0.20,0.30)	(0.80,0.70,0.50)	(0.85,0.20,0.30)	(0.60,0.50,0.70)
	$\mathring{A}_3$	(0.80,0.30,0.40)	(0.75,0.40,0.50)	(0.60,0.50,0.70)	(0.90,0.10,0.20)	(0.40,0.30,0.80)

	$\mathring{A}_4$	(0.70,0.50,0.60)	(0.90,0.10,0.20)	(0.70,0.60,0.60)	(0.70,0.50,0.60)	(0.65,0.55,0.65)
	$\mathring{A}_5$	(0.85,0.20,0.30)	(0.75,0.40,0.50)	(0.75,0.65,0.55)	(0.75,0.40,0.50)	(0.50,0.40,0.75)
$\mathcal{D}_2$	$\mathring{A}_1$	(0.85,0.20,0.30)	(0.80,0.30,0.40)	(0.70,0.60,0.60)	(0.90,0.10,0.20)	(0.80,0.70,0.50)
	$\mathring{A}_2$	(0.80,0.30,0.40)	(0.85,0.20,0.30)	(0.60,0.50,0.70)	(0.80,0.30,0.40)	(0.80,0.70,0.50)
	$\mathring{A}_3$	(0.90,0.10,0.20)	(0.70,0.50,0.60)	(0.65,0.55,0.65)	(0.90,0.10,0.20)	(0.70,0.60,0.60)
	$\mathring{A}_4$	(0.80,0.30,0.40)	(0.90,0.10,0.20)	(0.75,0.65,0.55)	(0.85,0.20,0.30)	(0.60,0.50,0.70)
	$\mathring{A}_5$	(0.85,0.20,0.30)	(0.80,0.30,0.40)	(0.80,0.70,0.50)	(0.85,0.20,0.30)	(0.40,0.30,0.80)
$\mathcal{D}_3$	$\mathring{A}_1$	(0.75,0.40,0.50)	(0.80,0.30,0.40)	(0.60,0.50,0.70)	(0.75,0.40,0.50)	(0.75,0.65,0.55)
	$\mathring{A}_2$	(0.75,0.40,0.50)	(0.75,0.40,0.50)	(0.60,0.50,0.70)	(0.85,0.20,0.30)	(0.65,0.55,0.65)
	$\mathring{A}_3$	(0.80,0.30,0.40)	(0.75,0.40,0.50)	(0.80,0.70,0.50)	(0.90,0.10,0.20)	(0.60,0.50,0.70)
	$\mathring{A}_4$	(0.80,0.30,0.40)	(0.85,0.20,0.30)	(0.75,0.65,0.55)	(0.80,0.30,0.40)	(0.80,0.70,0.50)
	$\mathring{A}_5$	(0.90,0.10,0.20)	(0.85,0.20,0.30)	(0.70,0.60,0.60)	(0.70,0.50,0.60)	(0.65,0.55,0.65)

Table 4
Evaluation to determine the experience levels of the Ɗ

Ɗ	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$	**Experience of Ɗ**
$\mathcal{D}_1$	(0.50,0.70,0.80)	(0.80,0.30,0.40)	(0.70,0.50,0.60)	(0.60,0.60,0.70)	(0.90,0.10,0.20)	0.38
$\mathcal{D}_2$	(0.90,0.10,0.20)	(0.60,0.60,0.70)	(0.50,0.70,0.80)	(0.80,0.30,0.40)	(0.70,0.50,0.60)	0.30
$\mathcal{D}_3$	(0.90,0.10,0.20)	(0.50,0.70,0.80)	(0.80,0.30,0.40)	(0.70,0.50,0.60)	(0.60,0.60,0.70)	0.32

Table 5
Defuzzified and then normalized attribute weights of the Ɗ

Attribute	**Aggregated weights**	**Aggregated weights are defuzzified**	**Prioritized weights of attributes**
$\hat{C}_1$	(0.83,0.21,0.34)	0.54	0.38
$\hat{C}_2$	(0.69,0.48,0.59)	0.12	0.08
$\hat{C}_3$	(0.70,0.47,0.57)	0.16	0.11
$\hat{C}_4$	(0.71,0.46,0.56)	0.18	0.13
$\hat{C}_5$	(0.80,0.29,0.42)	0.43	0.30

Step 4.1 to Step 4.4: Aggregate Table 3 with respect to experience levels of Ɗ based on Eq. (7), and the aggregated DM is presented in Table 6. Next, we use Eqs. (21) and Eq. (23) based on the prioritized weights of attributes and the aggregated results of $Q_{\dot{I}}^{S}$ and $Q_{\dot{I}}^{P}$ are shown in Table 7.

Table 6
Normalized aggregated DM

$\mathring{A}/\hat{C}$	$\hat{C}_1$	$\hat{C}_2$	$\hat{C}_3$	$\hat{C}_4$	$\hat{C}_5$
$\mathring{A}_1$	(0.85,0.19,0.30)	(0.80,0.30,0.40)	(0.70,0.58,0.61)	(0.83,0.24,0.35)	(0.79,0.68,0.52)
$\mathring{A}_2$	(0.84,0.22,0.33)	(0.82,0.25,0.35)	(0.70,0.57,0.62)	(0.84,0.23,0.33)	(0.70,0.57,0.62)
$\mathring{A}_3$	(0.84,0.22,0.32)	(0.74,0.43,0.53)	(0.70,0.57,0.61)	(0.90,0.10,0.20)	(0.59,0.43,0.70)
$\mathring{A}_4$	(0.77,0.36,0.47)	(0.89,0.12,0.23)	(0.73,0.63,0.57)	(0.79,0.32,0.43)	(0.70,0.58,0.61)
$\mathring{A}_5$	(0.87,0.16,0.26)	(0.80,0.29,0.40)	(0.75,0.65,0.55)	(0.78,0.35,0.45)	(0.54,0.41,0.73)

Table 7
Accumulated results

Alternatives	$Q_{\dot{I}}^{S}$	$Q_{\dot{I}}^{P}$
$\mathring{A}_1$	(0.81,0.34,0.40)	(0.81,0.34,0.45)
$\mathring{A}_2$	(0.79,0.33,0.43)	(0.78,0.33,0.50)
$\mathring{A}_3$	(0.79,0.28,0.43)	(0.74,0.28,0.55)

$\mathring{A}_4$	$(0.77,0.40,0.48)$	$(0.76,0.40,0.52)$
$\mathring{A}_5$	$(0.78,0.29,0.43)$	$(0.73,0.29,0.56)$

Finally, we utilize Eq. (24) by assuming $\mathbb{C} = 0.5$; as a result, we obtain accumulated value of $Q_{\ddot{I}}$, showcased in Table 8.

Table 8
WASPAS model

Alternatives	$\mathbb{C}\boldsymbol{Q}_{\ddot{I}}^{S}$	$(\mathbf{1}-\mathbb{C})\boldsymbol{Q}_{\ddot{I}}^{P}$	$\boldsymbol{Q}_{\ddot{I}} = \mathbb{C}\boldsymbol{Q}_{\ddot{I}}^{S} + (\mathbf{1}-\mathbb{C})\boldsymbol{Q}_{\ddot{I}}^{P}$
$\mathring{A}_1$	$(0.68,0.58,0.63)$	$(0.68,0.58,0.67)$	$(0.81,0.34,0.42)$
$\mathring{A}_2$	$(0.66,0.57,0.65)$	$(0.65,0.57,0.71)$	$(0.78,0.33,0.46)$
$\mathring{A}_3$	$(0.66,0.53,0.65)$	$(0.61,0.53,0.74)$	$(0.76,0.28,0.49)$
$\mathring{A}_4$	$(0.64,0.63,0.69)$	$(0.63,0.63,0.72)$	$(0.76,0.40,0.50)$
$\mathring{A}_5$	$(0.65,0.54,0.66)$	$(0.60,0.54,0.75)$	$(0.76,0.29,0.49)$

Step 5: Using Eq. (10), we calculate the score values of $Q_{\ddot{I}}^{S}$, $Q_{\ddot{I}}^{P}$, and $Q_{\ddot{I}}$ for each alternative, as presented in Table 7 and Table 8. Based on these score values, the ranking of each alternative is determined and showcased in Table 9.

Table 9
Each alternative score value and ranking

Alternatives	$\mathring{A}_1$	$\mathring{A}_2$	$\mathring{A}_3$	$\mathring{A}_4$	$\mathring{A}_5$	Ranking
Score of $\boldsymbol{Q}_{\ddot{I}}^{S}$	0.48	0.42	0.41	0.34	0.40	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$
Score of $\boldsymbol{Q}_{\ddot{I}}^{P}$	0.43	0.34	0.23	0.29	0.21	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_4 > \mathring{A}_3 > \mathring{A}_5$
Score of $\boldsymbol{Q}_{\ddot{I}}$	0.46	0.38	0.33	0.31	0.32	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$

Table 9 presents the overall ranking of alternatives, which varies across the TSFWA, TSFWG, and WASPAS approaches. Specifically, the ranking of TSFWA is $\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$ the ranking of TSFWG is $\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_4 > \mathring{A}_3 > \mathring{A}_5$, and the ranking of WASPAS is $\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$. In all these cases, $\mathring{A}_1$ emerges as the best alternative, representing the air quality health index in the TSF environment. However, the final decision depends on the consideration of Ɖ. The ranking behavior and alternative comparisons can be easily visualized in Figure 3.

Figure 3 illustrates the rankings of each alternative based on scores obtained using the TSFWA and TSFWG operators, as well as the WASPAS technique in the TSF environment. The graphical representation in Figure 3 consistently shows that $\mathring{A}_1$ is the best alternative, representing the air quality health index. Notably, the blue color in Figure 3 indicates the optimal option.

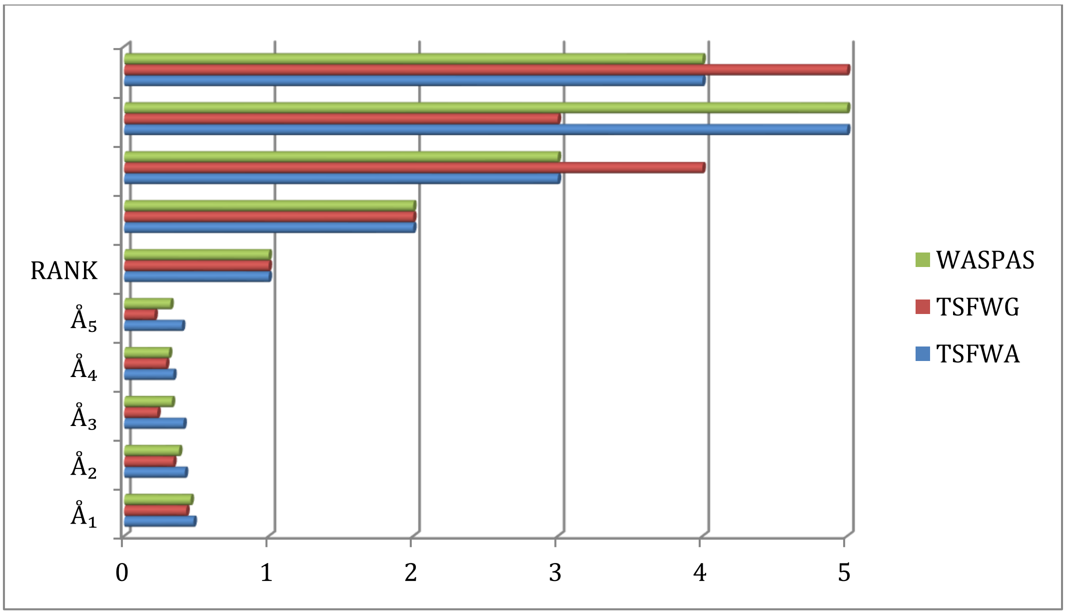

Fig. 3. Graphical Representation of Alternatives

4.4. Kendall's Coefficient of Concordance

In this subsection, we verify the level of agreement among the three ranking methods, TSFWA, TSFWG, and WASPAS, based on Kendall's coefficient of concordance. In this analysis, the three methods are treated as judges, and the five alternatives are the objects being ranked.

Step 1: Based on Table 9, the rankings obtained from each method are summarized in Table 10.

Table 10
Ranking DM construction

Alternatives	$Q_{Ï}^{S}$	$Q_{Ï}^{P}$	$Q_{Ï}$
$Å_1$	1	1	1
$Å_2$	2	2	2
$Å_3$	3	4	3
$Å_4$	5	3	5
$Å_5$	4	5	4

Step 2: The total rank for each alternative is calculated by summing its ranks across the three methods presented in Table 11.

Table 11
Sum of ranks of each alternative

Alternatives	$\mathcal{R}_{Ï}$
$Å_1$	3
$Å_2$	6
$Å_3$	10
$Å_4$	13
$Å_5$	13

Step 3: The number of ranking methods is denoted by $m = 3$ and the number of alternatives denoted by $n = 5$. Then, the average rank sum is given:

$$\bar{\mathcal{R}} = \frac{m(n+1)}{2} = \frac{3(5+1)}{2} = 9$$

Step 4: The squared deviation of each rank sum from the mean is computed and showcased in Table 12.

Table 12
Square deviation DM

Alternatives	$\mathcal{R}_{\text{Ï}} - \bar{\mathcal{R}}$	$(\mathcal{R}_{\text{Ï}} - \bar{\mathcal{R}})^2$
$\mathring{A}_1$	6	36
$\mathring{A}_2$	-3	9
$\mathring{A}_3$	1	1
$\mathring{A}_4$	4	16
$\mathring{A}_5$	4	16

Step 5: Calculate Kendall's coefficient of concordance, denoted by $\mathcal{W}$ and given as:

$$\bar{\mathcal{R}} = \frac{12\sum(\mathcal{R}_{\text{Ï}} - \bar{\mathcal{R}})^2}{m^2(n^3 - n)} = \frac{12(78)}{3^2(5^3 - 5)} = 0.87$$

Step 6: The value of Kendall's coefficient of concordance, $\mathcal{W} = 0.87$ obtained demonstrates that the agreement between the TSFWA, TSFWG, and WASPAS methods of ranking is very high. All three methods show that the alternative $\mathring{A}_1$ is the best choice**,** which demonstrates the robustness of the final decision in the TSF environment.

5. Sensitivity Analysis

In this section, we assess the effectiveness of our proposed approaches, defined in Eqs. (21), (23), and (24), by applying them to the data in Table 3. The results in Table 13 show that the ranking of the alternatives does not change significantly for different parametric values in the range 0.1 to 0.9. This consistency reflects their strength and reliability in the methods we propose for evaluating alternatives across different scenarios, making them potentially useful for air quality assessment and decision-making in practice.

Table 13
Score values and ranking of the sensitivity analysis

$\mathbb{C}$	$\mathring{A}_1$	$\mathring{A}_2$	$\mathring{A}_3$	$\mathring{A}_4$	$\mathring{A}_5$	**Ranking**
$\mathbb{C}_1 = \mathbf{0.1}$	0.44	0.35	0.26	0.29	0.23	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_4 > \mathring{A}_3 > \mathring{A}_5$
$\mathbb{C}_2 = \mathbf{0.2}$	0.44	0.36	0.28	0.30	0.26	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_4 > \mathring{A}_3 > \mathring{A}_5$
$\mathbb{C}_3 = \mathbf{0.3}$	0.45	0.37	0.30	0.30	0.28	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_4 > \mathring{A}_3 > \mathring{A}_5$
$\mathbb{C}_4 = \mathbf{0.4}$	0.45	0.38	0.31	0.31	0.30	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_4 > \mathring{A}_5$
$\mathbb{C}_5 = \mathbf{0.5}$	0.46	0.38	0.33	0.31	0.32	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$
$\mathbb{C}_6 = \mathbf{0.6}$	0.46	0.39	0.35	0.32	0.34	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$
$\mathbb{C}_7 = \mathbf{0.7}$	0.46	0.40	0.36	0.32	0.35	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$
$\mathbb{C}_8 = \mathbf{0.8}$	0.47	0.40	0.38	0.33	0.37	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$
$\mathbb{C}_9 = \mathbf{0.9}$	0.47	0.41	0.39	0.33	0.39	$\mathring{A}_1 > \mathring{A}_2 > \mathring{A}_3 > \mathring{A}_5 > \mathring{A}_4$

Table 13 reveals that alternative $Å_1$ consistently emerges as the best option across a range of parameter values. The sensitivity analysis also validates this conclusion, showing that, for all the parametric values, the alternative with the lowest air quality index is always $Å_1$. This uniformity displays the stability of our proposed method and demonstrates that $Å_1$ is a promising contender for implementation in decision-making situations. The robustness of the ranking also illustrates the potential of our approach to provide consistent, stable support for decision-making in complex, dynamic settings.

6. Comparative Discussion

We will explain why we would do this in the TSF environment and how we would do it for the problem in Section 4. We show that our proposed methodology is feasible and effective through the development of the comparison study. In the comparative argument, we can evaluate our performance and the strength of our argument, while leveraging the argument to solve a complex decision-making problem. Evaluating the results and comparing them with the methods will demonstrate the validity and applicability of our approach in real-world settings.

According to the operator of TSFWA, it is compared with previous averaging operators, such as the TSF Einstein hybrid weighted averaging (TSFEHWA) operator, the TSF weighted averaging interaction (TSFWAI) operator, and the TSF Hamacher weighted averaging (TSFHWA) operator. Table 3 displays the data ranking, and Table 14 provides a comparison view.

Table 14
Comparative discussion of the proposed averaging operator with existing approaches

Author	Operator	Parameter	Ranking of decision results	Best alternative
Reference [28]	TSFEHWA	Not consider	$Å_1 > Å_2 > Å_3 > Å_5 > Å_4$	$Å_1$
Reference [29]	TSFWAI	Not consider	$Å_1 > Å_2 > Å_3 > Å_5 > Å_4$	$Å_1$
Reference [30]	TSFHWA	$\phi = 0.50$	$Å_1 > Å_2 > Å_3 > Å_5 > Å_4$	$Å_1$
Proposed	TSFWA	$\mathbb{C} = 0.50$	$Å_1 > Å_2 > Å_3 > Å_5 > Å_4$	$Å_1$

According to the operator of TSFWG, it is compared with previous averaging operators, such as the TSF Einstein hybrid weighted averaging (TSFEHWG) operator, the TSF weighted averaging interaction (TSFWGI) operator, and the TSF Hamacher weighted averaging (TSFHWG) operator. Table 3 displays the data ranking, and Table 15 provides a comparison view.

Table 15
Comparative discussion of the proposed geometric operator with existing approaches

Author	Operator	Parameter	Ranking of decision results	Best alternative
Reference [28]	TSFEHWG	Not consider	$Å_1 > Å_2 > Å_4 > Å_3 > Å_5$	$Å_1$
Reference [29]	TSFWGI	Not consider	$Å_1 > Å_2 > Å_4 > Å_3 > Å_5$	$Å_1$
Reference [30]	TSFHWG	$\phi = 0.50$	$Å_1 > Å_2 > Å_4 > Å_3 > Å_5$	$Å_1$
Proposed	TSFWG	$\mathbb{C} = 0.50$	$Å_1 > Å_2 > Å_4 > Å_3 > Å_5$	$Å_1$

Based on the initiated AOs and the WASPAS method, TSFWA and TSFWG operators are compared with several former operators, including TSFEHWA and TSFEHWG, TSFWAI and TSFWGI, and TSFHWA and TSFHWG. The data ranking is presented in Table 3, and the comparison view is shown in Table 16.

Table 16
Comparative discussion of the proposed WASPAS method with existing approaches

Author	Operator	Parameter	Convex formula	Ranking of decision result	Best alternative
Reference [28]	TSFEHWA TSFEHWG	Not consider	Not consider	$Å_1 > Å_2 > Å_4 > Å_3 > Å_5$	$Å_1$
Reference [29]	TSFWAI TSFWGI	Not consider	Not consider	$Å_1 > Å_2 > Å_3 > Å_5 > Å_4$	$Å_1$
Reference [30]	TSFHWA TSFHWG	$\phi = 0.50$	Not consider	$Å_1 > Å_2 > Å_3 > Å_5 > Å_4$	$Å_1$
Proposed	TSFWA TSFWG	$\mathbb{C} = 0.50$	$Q_{Ï} = \mathbb{C}Q_{Ï}^{S} + (1-\mathbb{C})Q_{Ï}^{P}$	$Å_1 > Å_2 > Å_3 > Å_5 > Å_4$	$Å_1$

In Figures 4 and 6, we presented graphical representations of the score values and the ranking of alternatives, respectively.

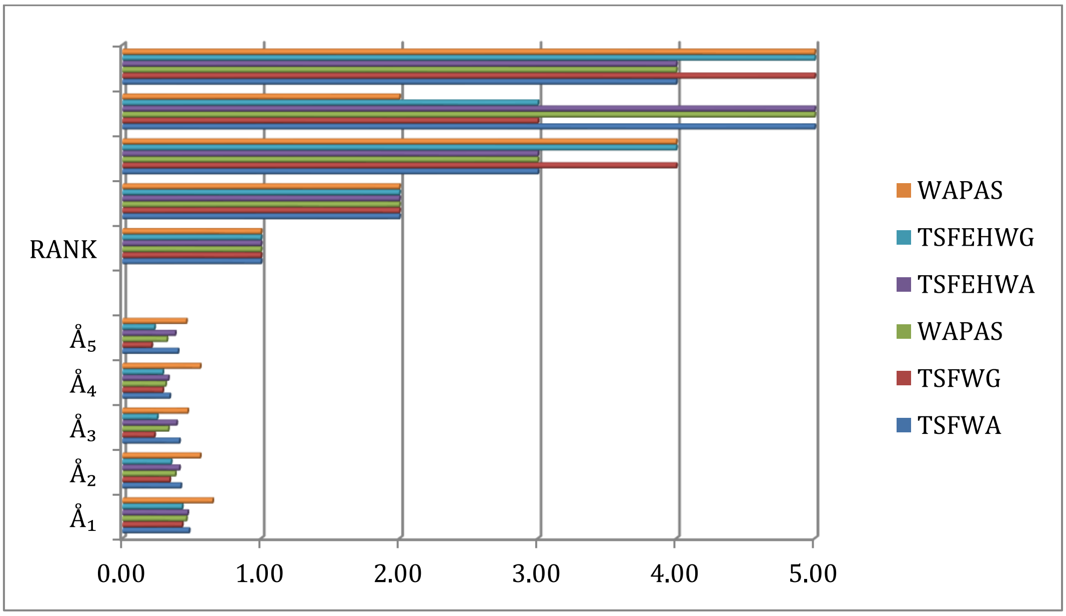

Fig.4. Geometrical representation of proposed work with existing work

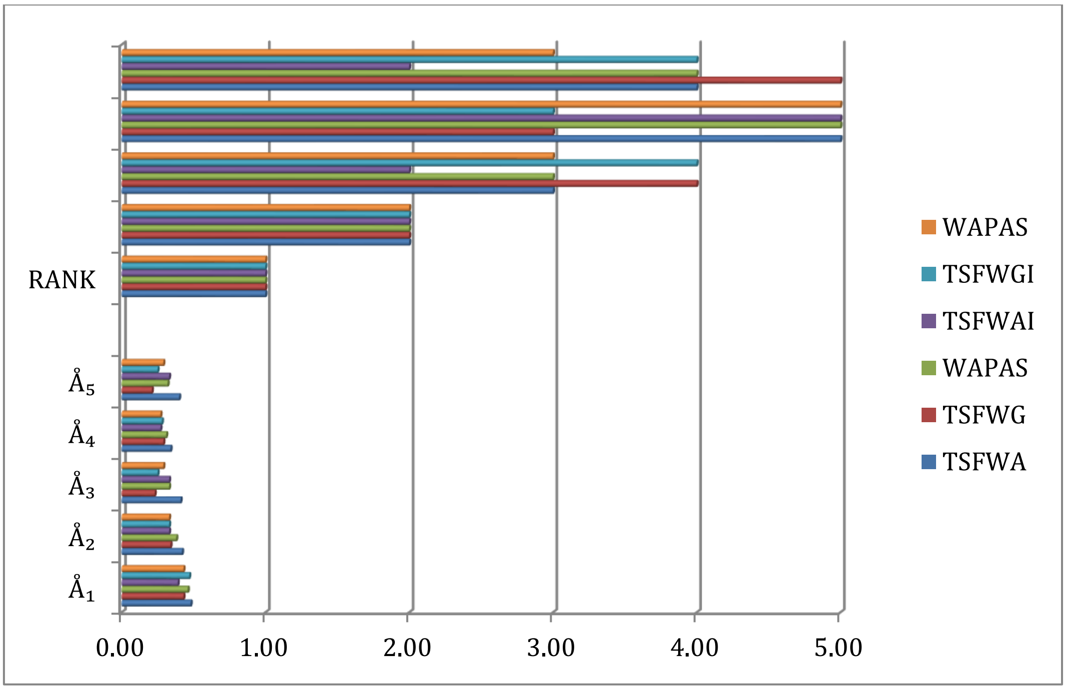

Fig.5. Geometrical representation of proposed work with existing work

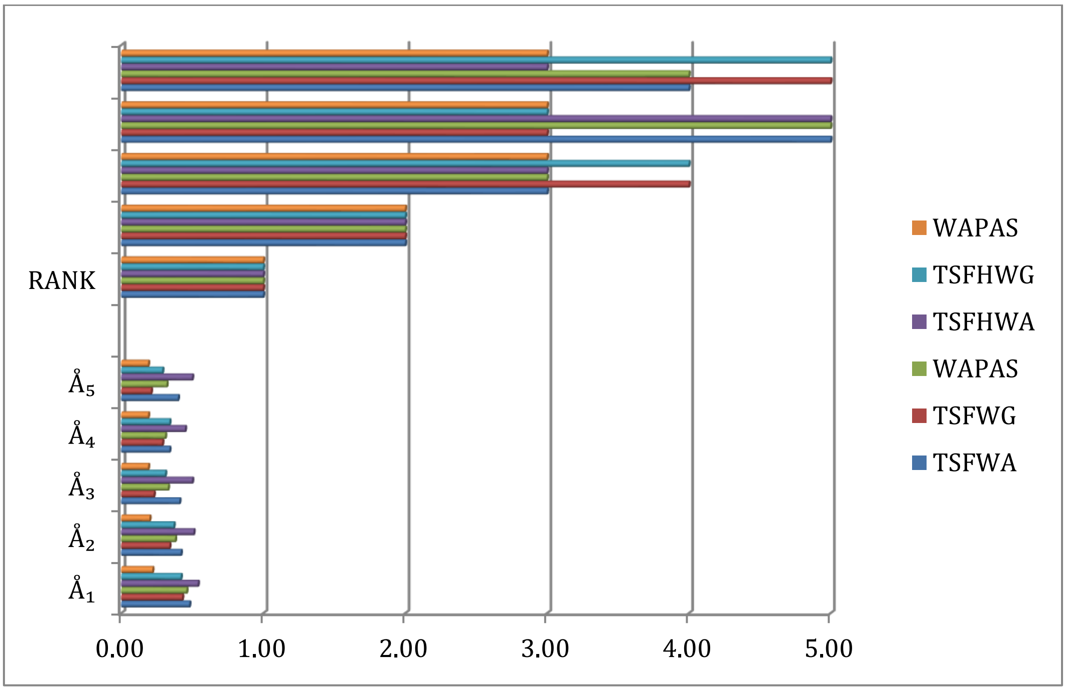

Fig. 6. Geometrical representation of proposed work with existing work

In Figures 4-6, the defined work is presented in Eqs. (21) and (23) are almost the same as the earlier AOs presented in [28], [29], and [30], which are utilized in this section. However, in the case of Eq. (24), the ranking order differs slightly when we use the earlier AOs presented in [28], [29], and [30], which are used in this section. Moreover, we observe during the comparison that $Å_1$ is the best alternative by taking any aggregation operator. However, we note that the outcome depends on the Ɗ consideration, the choice of operator, and the user's skill level.

7. *Conclusion*

In this article, we leveraged TSF information, a highly authentic and reliable concept within FS theory, with significant implications for decision-making applications. As a generalization of three-dimensional FS theory, the TSF context provides a substantive framework for complex decision-making. By combining the WASPAS method and the TSFS, we created an effective approach for resolving MAGDM problems. This methodology is very flexible and can manage relationships among different attributes depending on parametric values. We used numerical examples to illustrate the effectiveness of our approach, in which we determined the best alternative for each criterion. To ensure our model is reliable, we have performed a sensitivity analysis, assigning various parametric values and observing their effects on the alternative options. Additionally, the comparative section using available aggregation operators validated the superiority and suitability of our proposed method.

This research identifies important findings that can inform the development of decision-making processes in complex, uncertain environments. To determine the theoretical basis, first, TSFS and their basic laws, such as score values, accuracy values, and operators, are re-examined. In this context, applying the TSF environment to decision-making will be examined to compare options and establish the significance of attributes. It is followed by a normalization process to manage the beneficial and cost attributes in DM, ensuring consistent evaluation. The attribute weight priorities are considered and incorporated into the evaluation of MAGDM problems, where the WASPAS technique is suggested for using the TSFWA and TSFGA operators to combine the DM effectively. In this regard, a systematic approach and an algorithm are developed to solve MAGDM problems. The feasibility of the approach is demonstrated by applying it to the air quality index, which demonstrates its ability to handle uncertain, interrelated factors. Lastly, sensitivity and compatibility analyses are used to confirm the strength of the presented technique relative to known techniques. The research successfully addresses the problem of inherent uncertainty modeling and provides an evaluation support system that enables sound, effective decisions in complex, dynamic environments.

7.1. Limitations

The TSF information approach is superior because it is effective, operates within a close interval of 0 to 1, and does not restrict grades. By combining the WASPAS method with the TSF framework, we develop an advanced model capable of solving complex problems. However, the proposed model has some limitations, such as its reliance on information formatted according to Definition 1, which may not fit complex, circular, hesitant, or fuzzy information that may require other methods. In addition, the model requires non-zero attribute weights that sum to 1 to obtain precise results. Despite these restrictions, the suggested model makes a significant contribution to the FS theory framework, especially in the processing of TSF information.

7.2. Future Direction

Future research directions will involve the identification of new methods for applying the FS theory framework, such as the development of Einstein operators, Aczel-Alsina power operators, Archimedean Heronian mean operators, and Frank operators in circular and fuzzy complex environments. Besides, we aim to incorporate additional approaches, including CRITIC-WASPAS, CRITIC-MULTIMOORA, MABAC [35], CODAS [36] and MARCOS, to further enhance the value and usability of the proposed theory. In the future, we can extend the concepts to MAGDM methods for energy supplier selection based on [30] using the TOPSIS method, Frank AOs [31], Einstein AOs [32], the Archimedean Heronian mean operator [34], Spherical Fuzzy Bonferroni Mean [37] and q-Rung Orthopair Spherical Fuzzy sets [38].

Author Contributions

Zeeshan Ahmad wrote the abstract, introduction, WASPAS method algorithm, methodology, and case study. Kifayat Ullah built a background and sensitivity analysis. Zeeshan Ali built a comparative analysis and conclusion for the manuscript. All authors approved the manuscript.

Data availability

The datasets used and/or analyzed during the current study are available from the corresponding authors upon reasonable request.

Conflict of interests

The authors declare no conflict of interest.

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